

CONTENT ENRICHMENT IN MATHEMATICS TEACHING FOR THE  
SC/ST KEY PERSONS OF MEGHALAYA RELATED  
TO ELEMENTARY EDUCATION

FROM : 12TH - 16TH MARCH, 1993  
VENUE : S.C.E.R.T., SHILLONG.

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## P R E F A C E

It is observed that the students of North East States, especially of tribal states, seem very much scared of the study of Mathematics and Science subjects at School level. Some of the teachers who were appointed long back are under qualified <sup>and</sup> have not acquired efficiency and expertise for teaching Mathematics effectively. Even many schools have no science and mathematics teachers separately to deal with the subjects.

Above and all, the State of Meghalaya has developed new curriculum of teaching in Mathematics based on NPE. The contents are covered <sup>by NCERT books.</sup> The books published are up to date and contain many new topics which were taught previously in advance classes. These new topics and their delivery treatment are new to these serving teachers.

The teaching of mathematics at elementary level is more pathetic. At Primary Level even non-matriculate teachers are serving who have never come across with the new mode of mathematics teaching. With the skills and competency available with them they can not do justice with mathematics teaching, and if the elementary level teaching of mathematics is weak, the high school teaching can not be strong and thus can not deliver the desired level of competency in the students. So the very purpose is defeated.

Keeping the problems in mind, it was decided that let some efforts be made to orient a group of key persons related to mathematics teaching at elementary level. Thus the clients were selected from teacher educators/High School teachers/Head Masters of M.E.School whose services could be utilised by the State Government for their teacher training.

A training course entitled "Content Enrichment in Mathematics for the SC/ST teacher educators related to elementary education of Meghalaya" was mooted out. It was planned to train up 30 Key Persons. The planning resulted into the conduct of the training programme for five days from 12.3.93 to 16.3.93 at SCERT, Shillong with the following Chief objectives to be achieved:

1. to acquaint them with the importance of mathematics teaching.
2. to expose with new topics incorporated into various mathematics books at various levels of elementary classes.
3. to encourage them to prepare introducing lessons on new topics.
4. to discuss contents and methodology of mathematics at elementary level.

The programme was conducted for the short period but the participants enjoyed it very much. They expressed to extend it for some more days. The Education Minister of Meghalaya, Dr. Henry Lamin wished that such programme would have lasted for more days so that the participants could have been benefited to a large extent.

It can not be claimed that the set objectives for the programme were achieved but the facts remained that sincere efforts were made in this direction. If the participants have got some insight, incentives and motivation from the programme, certainly it would be a great satisfaction to the organisers.

M.M. Pandey,  
Field Adviser.

## A C K N O W L E D G E M E N T

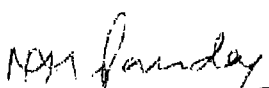
The Organisation of Training Programme on Content Enrichment in Mathematics Teaching for the SC/ST Teacher Educators/High School Teachers/Headmasters of M.E.School of Meghalaya was possible from 12.3.93 to 16.3.93 due to the continuous planning and faultless efforts made by the Office Staff. I am thankful to them.

The utmost sincerity and motivation shown by Sri.C.Volflang, Director, Mr.K.M.Syiam, Sr.Lecturer and Sri.R.Lyngdoh of SCERT remained a constant source for successful completion of training programme. It made me grateful to them.

I express my sincere and heartfelt gratitude to the external resource persons like Dr.B.K.Dev Sharma, Dr.P.K.Saikia, Department of NEHU, Dr.U.C.Vajpayee and Sri.H.K.P.Sinha, NMS Shillong and Prof.Man Mohon Singh, Head, Centre for Science and Mathematics, NEHU, Shillong for their eloquent exposure on various aspects of Mathematics teaching.

Last but not the least, the office is highly grateful to Prof.K.C.Panda, Principal, Regional College of Education, Bhubaneswar who promptly deputed Dr.V.Rao as expert to help in the conduct of the training on our request. Dr.Rao took much pain in looking after day to day activities of the course. I felt delighted as well as grateful to Dr.Henry Lamin, Minister of Education, Govt.of Meghalaya who made it convenient to address the participants for half an hour sparing time from his busy schedule.

At the end, I am not less thankful to the typist, psychostylist and others who helped immensely in getting out the report published in this form with the hope that it would help the teachers and students to a certain extent.

  
( Dr.M.M.Pandey )  
Programme Director.



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2. to discuss the difference between old course and new course.
3. to discuss the historical development of mathematics in and outside of India.
4. to discuss the topics incorporated in mathematics books from Class I to VII.

5. to expose the teachers with the teaching of modern mathematics and explain its importance.
6. to discuss applicable mathematics such as set theory, number system, equation, geometry etc.
7. to evaluate their knowledge in mathematics teaching up to the level of I, II, III.

8. to help them in preparing a lesson on contents as how it should be introduced in classroom situation.
9. to discuss as what method of teaching they are using in classroom situation at elementary level.
10. to expose them to other probabilities of teaching mathematics.

Further, Dr. Pandey sought suggestions from the participants as what they wanted to add more in the time table already prepared for the duration. The participants were happy with the time schedule and expressed that something might be added afterwards if there was any need-felt aspects of teaching.

Dr. Pandey requested them to feel free to ask if they find some difficulties to understand. They should also buy some NCERT books for reference. Their participation would help them in fulfilling the objectives of the training programme.

Dr. H.C. Vajpayee began his lecture with the definition of a linear equation in general and a linear equation in one variable in particular. He emphasised the need to understand the techniques of solving linear equation -

Any equation which can be put in the form of  $ax + b = 0$ ,  $a \neq 0$  is a linear equation in one variable. Dr. Vajpayee listed the various rules followed while solving a linear equation.

P R O C E E D I N G S

FORENOON  
12.3.1993.

The Content Enrichment in Mathematics for Key persons dealing with Elementary Education conducted by Field Adviser Office, NCERT, Shillong for five days from 12.3.93 to 16.3.93 was inaugurated by Dr. U.C. Vajpayee, Na- vodaya Vidyalaya Samiti, Regional Office, Shillong. This training programme for Meghalaya Mathematics teachers was presided by Sri. A.G. Momin, Dy. D.P.I., Government of Megha- laya. The venue was the building opposite to SCERT, Shillong for, NCERT, Shillong while welcoming the guests and parti- cipants throw brief light on the functioning of NCERT and its constituents units - NCES/FACS. He explained the im- portance of Mathematics teaching at elementary level.

Dr. Pandey outlined the objectives of the course being organised. As the elementary teacher educators were the facilitator and trainers of the teachers to teach at elementary classes, so it was pertinent to re-orient them. The High School Mathematics teachers do have links with elementary mathematics, thus it was felt necessary to re- work a few of them also as they persons whose services can be utilised for further training to the State teachers. Meghalaya has developed the new curriculum and started teaching Mathematics books published and prepared on NEP by NCERT which cover their syllabus. These books are stuffed with all the topics and ingredients of mathe- matical knowledge. The Mathematics teachers teaching old courses find difficulties to teach new courses as they need training and orientation in the new dimension of the subject. The feelings and observations lead to design this training programme for five days though it would be so short a period that many things could not be discussed.

Dr. Pandey outlined the following objectives of the course which would be tried.

1. to explain the importance and objectives of mathe- matics teaching at elementary level.

He discussed the techniques in solving equation of the type

$$\frac{ax + b}{cx + d} = g$$

which could be reduced to linear equation. He explained the various steps involved in solving a day to day/<sup>such</sup> problem by converting it in to a linear form. AFTERNOON SESSION

Mr.F.M.Syiem elaborated the concept of number which is associated with quantity. He explained the role of number in the mathematical world. He distinguished between number and numerals. There should be no confusion between these terms .. he stressed. He discussed the Hindu, Arabic and Roman system of numeration. He explained with example the terms (i) place value and (ii) face value of a digit in numerals. He introduced the natural numbers and discussed the concept of successor and predecessor in the set of natural numbers. He established that there was no greatest natural number.

Mr.R.Lyngdoh explained the concept of set with simple and common examples. He discussed the meaning of set, element and the symbol E. He defined the empty set as the set having no element stands to represent the empty set. He narrated that two sets were said to be equal if they had the same elements. He defined further the sub-set and showed that  $\emptyset$  is a sub-set of every set. If a set has 'n' elements it would have  $2^n$  sub-set, he clarified.

Dr.Kameswar Rao throw light on the necessity and the concept of set and said it was George Cantor who first ventured to define set. He stressed the need to take the basic terms as undefined by mentioning the famous Russels' Paradox. He distinguished between set and well defined set. During his exposition, Dr.Rao explained the importance of teaching of modern mathematics in schools. The new treatment of mathematics teaching has opened a new vista of acquiring knowledge by teachers and students.

FORENOON SESSION  
13.3.1993

Dr.M.M.Pandey, reminded the participants of the objectives of teaching Mathematics at elementary level.

He emphasised the need of Mathematics, its inter-relationship with other branches of teaching subjects. He emphasised the need to make very clear in the young minds the power of mathematics. He maintained that mathematics develops analytical thinking and reasoning and helps in taking decisions accurately. Emphasising the importance, he examined the utilitarian, practical, ethical and disciplinarian values of mathematics. He noted the following chief objectives of teaching mathematics at elementary level.

1. to develop the reasoning and thinking capacity of the children.
2. to develop a right type of habit and attitude in the children.
3. to foster a definite sense of proportion to regulate the day to day life with a sense of responsibility.
4. to develop inductive, deductive, analytical and synthetic method of teaching mathematics.
5. to discuss the importance of getting the mind exercised and drilled through mathematical calculation for disciplinary and practical values.
6. to create a logical and practical mind for all practical purposes.

Dr.K.Rao wished that the teaching learning process to be child centred. His main thrust of the speech was to remove the fear of Mathematics study from the minds of young children. He emphasised the need for the mathematics teacher to be more affectionate and loving to the students. Through their such act they would be able to grow self confidence in the children. He discredited the general statement, "Mathematics is for only intelligent children" by giving the example of Pascal whose father advised the teacher not to tax the boy's mind with mathematical hazards because the teacher had asked Pascal, who was considered to be very weak in childhood, to write a complete treatise on conics at the age of 16 years. Dr.Rao emphasised to practise activities in the learning of Mathematics. He listed a number of activities which could be given to the students. He discussed the learning by discovery and the advantages of discovery method.

Dr. M.M. Singh outlined the importance of learning mathematics which helps in getting the knowledge of all disciplines. He stressed the importance of evaluation in teaching mathematics and narrated to evolve or design to evaluate the teaching-learning objectives. He gave the background of measurement and evaluation and its changing pattern and techniques in the prevailing situation. The conduct of evaluation in mathematics leads to the improvement of teaching learning process to a great extent - he observed.

All guardians including teachers are interested to know the outcomes of teaching, achievement and performances of their children. Prompt and regular evaluation fulfills their desires. It helps them in deciding their future course of action. Dr. Singh continued his lecture on the techniques of evaluation and its needs. He displayed a set of aids helpful in explaining the concept of triangle and square to the young children. Through exhibiting simple activities he proved that by using them teaching learning process could be made interesting, more fruitful and effective. Dr. Singh talked about the comprehensive and continuous evaluation. He explained unit tests which were used commonly in public schools these days.

#### AFTERNOON SESSION

Dr. Henry Lamin, Minister of Education, Govt. of Meghalaya made himself available to observe the proceedings of training programme. Being educationist to the core participated the teacher educators/teachers of his state. Dr. Pandey explained the NCERT and its functioning and duties towards the states. He also narrated the background of organising such training programme in the backdrop of the adoption of new syllabus by his state and NCERT mathematics books from this year.

Dr. Lamin showed his nobleness on the conduct of such programme. He stressed the need and importance of training in teacher's life. He emphasised that a teacher is he who remains a learner/student through out his life. He should learn every minute and refresh his memory and enrich his mind with the new happenings.

place in the multi-dimensions of the universe.

Further, he expressed that mathematics was the key to all knowledge. So its practice makes a person versatile genius, wise and logically stable and disciplined. A mathematics teacher is supposed to be a man of few words who believes in doing activities and knows to get the results by applying his mind. In the end he thanked the participants and the organisers. He requested the NCERT to organise such more programmes in Science and Mathematics for the benefit of Meghalaya teachers.

Dr. Pandey offered his heartiest thanks to Dr. Lambi who attended the programme within short notice and blessed the teachers.

Mr. Syiem explained the inadequacy of the Natural number system and called for the extension of the set of rationals. He discussed the place value in the decimal notation and related it to kilo, hector, meter etc. Prime Composite and unique natural numbers and their definitions were discussed. The participants took active interest in the proceedings and helped Mr. Syiem to reach the destination of his discussion.

Mr. Lyngdoh continued his lecture with the representation of a set in Roster and set builder form. He defined union and intersection of sets and explained with simple examples. He introduced Venn-diagrams as tools to represent sets to display various relations between sets. Universal set and complement of a set w.r.t. a given universal set were discussed. Giving examples, he demonstrated the commulative and Associative properties of union and intersection of sets. He established the fact that intersection is distributed over union and vice versa.

FORENOON  
14/3/1993.

The Participants, in the first half were given task to prepare a model lesson plan on any Mathematics topic which they like. They prepared the lessons of their choice and submitted to the Programme Director which would go in the report.

Dr. Rao began his lecture by giving definitions of point, line and plane as stated by Euclid in his elements. He emphasised the need to take these terms as  $\overleftrightarrow{AB}$ , the basic undefined terms. He said line is represented by a two sided arrow  $\overleftrightarrow{AB}$  and notation is  $AB$ . He defined the terms ray, line segment, angle, collinear points, degree, measure of an angle, triangle, median, altitude of a triangle and discussed concurrent lines, centroid, orthocentre and incentre. The concept of parallelism between lines was explained. Dr. Pandey helped Dr. Rao in defining and discussing many preliminary geometrical concepts.

Dr. Pandey assisted Dr. Rao in defining and explaining many preliminary geometrical concept which are given new treatment. The importance of teaching geometry was the sole points of discussion. The participants put their views forward and sought various clarifications.

FORENOON  
15/3/1993

Dr. P.K. Saikia began his lecture with stating the Natural numbers and said that  $\mathbb{N}$  is also known as the set of counting numbers. He said the Italian Mathematician Peano had formulated the axioms on Natural numbers. He explained the principle of induction of Mathematics with the help of some striking examples. He told though addition in commutative and associative  $\mathbb{N}$  had no addition inverse and additive identity thus there was a need to invent  $-1, -2, -3, \dots$ . He explained  $\mathbb{Z} = \{0, +1, +2, \dots\}$  had a beautiful structure but still there was no inverse for multiplication.

He discussed in detail

- (i) The fundamental theorem of Arithmetic
- (ii) Euclid's theorem as the number of primes
- (iii) Goldbach's conjecture

He explained the inadequacy of  $\mathbb{Z}$ ,  $\mathbb{Q}$  the rationals,  $\mathbb{R}$  the reals and thus the need to extend the number system to  $\mathbb{C}$ , the complex numbers.

Dr. Sinha discussed the equations of motion. He derived the following equations Mathematically

- (i)  $s = ut$  (Uniform velocity)
- (ii)  $v = u + at$
- (iii)  $u^2 = u^2 = 2as$
- (iv)  $S = ut + \frac{1}{2} at^2$
- (v) The distance travelled in the  $n$ th sec.  
 $= u + \frac{a}{2} (2n-1)$

He derived the corresponding equation for the freely falling bodies and the body projected upwards.

The value of  $g$ , the acceleration due to gravity at a height  $h$  from the surface of earth can be derived as

$$g = \frac{M}{(R+h)^2}$$

Where  $M$  is the Mass of the earth and  $R$  the radius of earth.

Dr.Dev.Sharma began his lecture with the remark "Mathematics is the queen and servant of Science". He defended his claim by quoting with several examples. He described the development of Mathematics very systematically. He said though Euclid is known as the father of geometry, it was Thales who first coined the word geometry. He discussed the Mathematics during vedic period and outlined the need for knowing the Vedic Mathematics by the present generation. He mentioned the contribution of Arya Bhatta, Bhaskaracharya, Ramanujan and Hardy to the world of Mathematics. He briefed the participants of Non-euclidean geometries too. He said that Mathematical truths were conditional.

#### AFTERNOON SESSION

Mr.Syiem began his lecture with the question how can you prove that  $-ve \times +ve = -ve$ ?

He established the following facts with the help of observations of the following patterns:

- (i)  $-3 \times 5 = 3 \times -5 = -15$   
 and in general  $+ve \times -ve = -ve$   
 $-ve \times +ve = -ve$
- (ii)  $-4 \times -5 = -5 \times -4 = 20$   
 and in general  $-ve \times -ve = +ve$
- (iii)  $3 \times 0 = 0 \times 3 = 0$   
 and  $0 \times$  any number  $= 0$

The participants took full interest in the discussion. He emphasised the need to know the division which was followed in computing the H.C.F. of two numbers. He explained the concept of equivalent fractions with the help of the number line. Further, Mr. Syiem highlighted the role of equivalent fraction in addition and comparison of rationals.

FORENOON  
16/1/1993.

Dr. Dev. Sharma discussed the very merable points in Mathematics, viz the application of mathematics. He told the participants that in loss and profit, percentage, numeration fraction etc. are commonly used in the day-to-day life in which literate or illiterate both are involved. He gave one practical example to prove his point as "If A exceeds B by 20% by what percent loss A is than B?". He explained why the answer was not 20%. Further he discussed the schematic method of solving such problem. He outlined the needs of percentage in (i) S.I. (ii) C.I. (iii) Profit and loss (iv) stocks and shares and (v) statistics. Briefly, he discussed the basic concepts of Geometry and its teaching at elementary level also.

Dr. Dev. Sharma explained the division method which is followed in computing the H.C.F. of two numbers. He highlighted the role of equivalent fractions in addition and comparison of rationals.

Dr. Rao and Dr. Pandey shared the class together and thus made the discussion lively. Dr. Rao, gave definition of an algebraic expression. He distinguished between variables and constants. Giving suitable example he explained various algebraic expression and defined a polynomial in one variable. He discussed further the characteristics of a polynomial viz. nature of the co-efficients degree of the polynomial and co-efficient of a given variable. He narrated the terms like monomial, binomial and trinomial. He derived linear and quadratic polynomials in one variable and their general form. The Zero of a linear polynomial and the difference between the zero and the zero of a polynomial could find place in his discussion.

Dr. Pandey intervened in the discussion and tried to find out the common mistakes committed by the students such as in writing +, x etc.

Continuing his lecture Dr. Rao outlined the need of activity method of learning mathematics in the classroom. He talked about designing activities in mathematics at various levels. He tried a sample of questions to check the mental ability and alertness. He listed a number of activities to generate interest in the teaching-learning process. Organising quiz competition, framing out mathematics activities, construction of Magic square etc. were helping a lot in creating a proper atmosphere in teaching mathematics. Dr. Rao asserted.

Dr. Wolflang spoke on the status of mathematics teaching in Meghalaya. He said the girls were not given adequate opportunities to learn mathematics earlier as the parents and teachers thought, girls could not learn mathematics effectively. It is only recently that mathematics has been made compulsory for all upto tenth standard. He lamented that previously the teachers never allowed the students to enjoy the learning of mathematics due to various misunderstanding.

Many of tribal students were failing in the subject not due to their inherent weaknesses but due to faulty method of teaching. They are frightened also. Dr. Wolflang observed. To avoid all these ills, he called upon the teachers to (i) inculcate interest in Mathematics-learning (ii) make the teaching child centred (iii) Orient themselves in activity based teaching and (iv) organise mathematics quiz, club and open competition for ensuring greater participation of students in mathematics learning.

To test the achievement of the participants and thus the competency acquired it was needed to construct some evaluating test in the end of formal orientation. After administering 30 no. of questions which covered mostly the topics covered received a good response from the participants which was a consolation to the organisers. The participants enjoyed the questionnaire fully.

AFTERNOON SESSION

Dr.C.Wolflang, Director SCERT was the Chief Guest in the valedictory function at the end of the programme.

Dr.M.M.Pandey presided over the function.

Dr.Pandey welcomed the guest and the participants. He presented a brief account of the activities conducted during the 5 days course. He touched upon the core of objectives set for the programme and hoped that those were achieved by and large, if not hundred percent. He expressed his happiness about the sincere efforts made towards the success of the programme as it was atleast able to motivate the teachers to devote their concentration on understanding the subject and then do the justice with the profession. Dr.Pandey requested the participants to be innovative in teaching as the children were in formative period and they were the sole masters.

Some of the participants were requested to offer their comments about the programme conducted. Mrs.H.Horoo thanked the organisers who conducted such programme in the interest of elementary level teachers. But she felt duration was too short. Sri.R.F.Thankhiew expressed his gratitude to the NCERT in general and the Field Adviser in particular for conducting programme in Meghalaya which had adopted a new syllabus for teaching.

Mr.K.M.Syiem, Sr.Lecturer and Mr.L.Lyngdoh, lecturer of SCERT also recorded their words of appreciation for involving them as resource persons in the programme. They felt their teachers needed to be exposed to a large extent in contents. In his address, Dr.Wolflang expressed heartfelt congratulations to NCERT and Field Adviser Office to conduct various useful training programmes for the teachers of Meghalaya. Dr.Rao from Regional College of Education, Shubanswar proposed the vote of thanks.

## NUMBER SYSTEMS - A QUICK SURVEY

P.SAIKIA  
MATHEMATICS DEPTT.  
NEHU, SHILLONG.

This lecture aims at discussing the various number systems one comes across at the school level mathematics. Some of the material covered may not be directly useful for or relevant to the school mathematics. But one hopes that the extra materials will make teachers more confident while discussing such system. It is also hoped that this lecture will be useful to those who want to convey the pleasures of doing mathematics to school children.

We start with the set of natural numbers  $N$ . One feels comfortable with these numbers because they are natural enough in the sense that they are used in counting real, concrete objects. At the school level, these numbers and their properties are assumed and rightly so. However, it should be pointed out that these numbers can be developed and their properties proved from the more basic and "natural" objects such as sets and mappings. (Such a development starts with Peano's axioms, a set of self-evident statements about a nonempty set with a "successor" mapping). In any case, we can list two important set theoretic properties of  $N$  such as :...

- (i) Every natural number has a successor which is different from the one with started with; so  $N$  is an infinite set,
- (ii) The Principle of Mathematical induction is available to prove results about  $N$ . The Principle says, "suppose we have a statement  $P(n)$  for every natural number  $n$ . If  $P(1)$  is true and if  $P(k+1)$  is true whenever  $P(k)$  is true, then  $P(n)$  is true for every  $n$ ". This principle is useful in diverse situations, e.g. in proving statements like

$$(A) \quad 1+2+\dots+n = \frac{n(n+1)}{2}$$

$$(B) \quad 1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

We can show, an example <sup>as</sup> how to prove (A). Now

$$P(n) : 1+2+\dots+n = \frac{n(n+1)}{2}$$

So  $P(1)$  is  $1 = \frac{1(1+1)}{2} = 1$  so that  $P(1)$  is true.

Assume

$$P(k) : 1+2+\dots+k = \frac{k(k+1)}{2}$$

is true.. Now

$$\begin{aligned} 1+2+\dots+k+(k+1) &= \frac{k(k+1)}{2} + (k+1) \\ &= (k+1) \left[ \frac{k}{2} + 1 \right] \\ &= \frac{(k+1)[(k+1) + 1]}{2} \end{aligned}$$

Showing  $P(k+1)$  is true. So by the principle  $P(n)$  is true for every  $n$  i.e. (A) is true.

$N$  has two natural "binary" operation namely, addition and multiplication (but subtraction and division are not as  $N$  is not "closed" with respect to these operations). It simply means the result of subtraction or division of two natural numbers need not be a natural number again). The properties of  $N$  with respect to addition and multiplication are

Commutative	$a+b = b+a$	$ab = ba$	} (x)
Associative	$(a+b)+c = a+(b+c)$	$(ab)c = a(bc)$	
Identity	None	$a.1 = a = 1.a$	
Distributive	$(a+b)c = ac+bc$	$a(b+c) = ab+ac$	

One will think that such a simple structure such as  $N$  will be very easy to understand. But nothing can be farther from the truth.  $N$  is as mysterious and complicated as anything can be. Infact, a most extensive branch of mathematics known as the theory of number mostly deals with  $N$  only.

Let us look at some of the beautiful facts about  $N$  concerning the primes. The primes are the building blocks of the natural numbers as clear from the following result known as the Fundamental theorem of Arithmetic : "Every natural number  $> 1$  can be written as a product of primes. Such a product is unique upto the order of the factors". One uses this result e.g. in finding lcm and gcd of natural numbers, or in finding square roots etc. But it has other deep and interesting applications. We shall see one when we consider numbers like  $\sqrt{2}$  and  $\sqrt{3}$  later. At the moment we shall use it to prove the following nontrivial result.

Them (Euclid); The number of primes is infinite.

Proof: (The method of proof employed is also due to the Greeks; one assumes the result is false and then derives a contradiction). Suppose not. Let  $p_1, p_2, \dots, p_t$  be all the primes. Consider  $N = (p_1 p_2 \dots p_t) + 1$ . Clearly  $N$  is a natural number  $\neq 1$ . So by the F.T.A., it follows that  $N$  is a product of primes from among  $p_1, \dots, p_t$  i.e. there is at least one  $p_i$  such that  $p_i | N$ . Since  $p_i$  divides the product  $(p_1 p_2 \dots p_t)$  also, it follows from the definition of  $N$  that  $p_i | 1$ , which is a contradiction. Hence the theorem is infinite.

This theorem is surprising because there is nothing in the definition of a prime that suggests anything like this. This is also the beginning of long list of beautiful results about the prime. We sample a few:

Goldbach's Conjecture: Every even natural number  $\geq 2$  is a sum of two primes. (This is not a result. This is a long-standing conjecture).

Mersenne numbers: Primes of the form  $2^p - 1$  are known as Mersenne where  $p$  is a prime. It is known that  $2^p - 1$  is prime for  $p=2, 3, 5, 7, 13, 17, 19, 31, 61, 127, 237$ . The question of which  $2^p - 1$  are prime for other primes  $p$  is not settled.

Twin primes: These are pairs of primes differing by 2. For example; 3 and 5; 5 and 7; 11 and 13; 29 and 31 etc. The questions that whether twin primes are infinitely many is not settled.

After this sample of curious facts about primes, let us consider the integers  $\mathbb{Z}$ . We are forced to consider the integers as subtraction in  $\mathbb{Z}$  leads to negative integers and Zero. Another way of describing the situation is: To have solutions of equations of type  $x+n=0$  where  $n \in \mathbb{N}$ , we want the integers. Two points to take note; the first is  $\mathbb{N} \subset \mathbb{Z}$  i.e.  $\mathbb{N}$  is a proper subset of  $\mathbb{Z}$  and the second is that with respect to addition of multiplication the integers apart from the properties listed in (\*) satisfy a few more:

Identity,  $a+0 = a = 0+a$

(Additive)

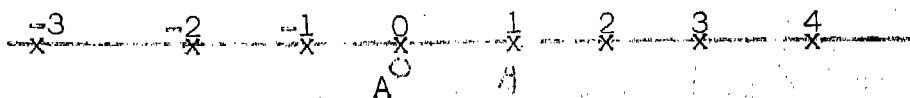
Inverse	for every a, there is a b such that $a+b = 0 = b+a$ (b is infact -a)	no such property w.r.t. multiplication.
---------	---	--

Note that  $\mathbb{Z}$  is still not "closed" w.r.t. division (though w.r.t. subtraction it is). We have to enlarge  $\mathbb{Z}$  to the rationals  $\mathbb{Q}$  in other words to have solutions of  $ax+b=0$  for a and b integers,  $a \neq 0$ .  $\mathbb{Q}$  also has addition and multiplication and apart from all the properties that  $\mathbb{Z}$  has it has the property that for every nonzero rational a, there is one b such that  $ab=1=ba$  (existence of multiplicative inverse). We also have  $\mathbb{Q}$  also has nice "order" relation (i.e. we can say whether one is bigger than the other).

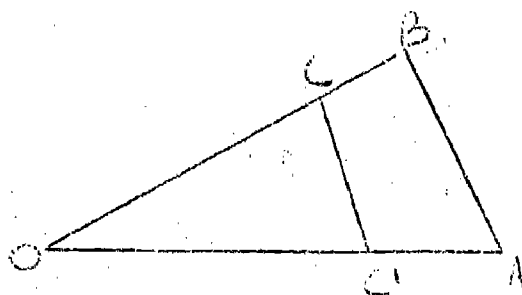
The Properties are:

- 1) Given  $a, b \in \mathbb{Q}$ , one and only one of the following holds;  
 $a < b$ ,  $a=b$ ,  $b < a$ .
- 2)  $a < b$ ,  $b < c \Rightarrow a < c$
- 3)  $0 < a$ ,  $0 < b \Rightarrow 0 < ab$
- 4)  $a < b \Rightarrow a+c < b+c$

We will be forced to consider a number system bigger than  $\mathbb{Q}$  once we wish to use numbers in geometry. In this case our requirement is that every line segment must have a number associated with it, namely, its length, and conversely corresponding to every number there must be a line segment ! To consider all line segments at one time we introduce a line indefinitely stretching in both directions.



We fix an origin o, a point of reference on the line and a line segment OA to fix the unit length 1. Hence OA has length 1; or, 0 corresponds to number 0 where as A correspond to 1. Then it is an easy matter to find points which correspond to 2,3,4,.....etc as well as to -1,-2,-3.....etc. How to find points corresponding to rational number? Well, one uses the ratios of sides of triangles. For example, to find corresponding to  $3/4$  we



Let  $OC$  be of length 3. Let  $C'$  be the point in  $OA$  such that  $CC'$  is parallel to the base  $BA$ . Then length  $OC' = 3/4$  i.e.  $C'$  corresponds to  $3/4$ . In this manner, we can associate all the rational numbers to various points on our line. After doing that, are there any point left on the line which do not correspond to any of the numbers we have considered so far? To see the complexity of this question consider the following: Let  $x$  and  $y$  be rationals such that  $x < y$ . Then  $\frac{x+y}{2}$  is a rational and  $x < \frac{x+y}{2} < y$ . Thus between two rationals there is one and hence there are infinitely many (as the same argument says there is a rational between  $x$  and  $\frac{x+y}{2}$  and so on and so forth). Thus it seems that once we "plot" the rational numbers on the line, there will not be any gaps left. But that is not true - there are points on the line which do not correspond to any rational (and hence to any integer or natural number). There is a line segment whose length is  $\sqrt{2}$  - namely the hypotenuse of a rightangled triangle whose both sides have length 1. (Use Pythagorus theorem).

Thm :  $\sqrt{2}$  is not rational

Pf: Suppose it is rational. Write  $\sqrt{2} = a/b$  where  $a$  and  $b$  are integers having no common factor. Thus  $2b^2 = a^2$ . So  $2/a^2$  i.e. the prime factorisation of  $a^2$  contains the prime 2. We conclude that the prime factorisation of  $a$  also contains 2 (otherwise the uniqueness part of the Fundamental theorem of Arithmetic is violated). In other words  $2/a^2$  means  $2/a$  or  $2^2/a^2$ . Going back to  $2b^2 = a^2$  we see that  $2/b^2$ , so  $2/b$ . Thus 2 is a common factor of both  $a$  and  $b$ , a contradiction. Hence theorem  $\square$  (similarly one can prove that  $\sqrt{p}$  for a prime  $p$  is not rational).

Hence we must add more numbers to  $\mathbb{Q}$  in case the members of the resultant set and the points on our line are in one-to-one correspondence i.e. to each number there is a point on the line and then each point corresponds to a number. This enlarged set of real number  $\mathbb{R}$ . (In terms of equations, we can say that we need to go from  $\mathbb{Q}$  to  $\mathbb{R}$  as equations like  $x^2 = 2$  have no solutions in  $\mathbb{Q}$ ). The real numbers, it turns out, with respect to addition and multiplications have the same properties as  $\mathbb{Q}$  does.

Are there number systems bigger than  $\mathbb{R}$ ? e.g. to solve equations like  $x^2 + 1 = 0$ , we have to enlarge the real numbers to complex numbers. Just as reals and points on a line are in one-to-one correspondence, the complex numbers and points on a plane are in one-to-one correspondence. We end our quick survey by pointing out that in a sense we do not have to go beyond the complex numbers for the Fundamental theorem of Algebra assures us that any (polynomial) equation with complex coefficient will always have solutions in the set of complex numbers only!

## INTRODUCTION TO SET THEORY

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In Mathematics, as in everyday life we are often concerned not with a single object but with a collection of objects. For example, we hear about and speak of a collection of paintings, a row of seats, a herd of cattle, a crowd of people, and a company of soldiers. Some collections are quite small, such as a pair of shoes or a pair of ear-rings, while other collections may be very large. The collection of cars on the roads during a Saturday afternoon or all the fish in the sea will both be large, for our idea of size will clearly depend on the number of objects in the collection. It would, of course, be an advantage if we use one word only to indicate a collection or gathering, instead of many available, such as row, herd, crowd and company. Each collection is an example of what we call a set, a word we frequently use in this sense anyway - a dinner set, a set of chessmen, and so on.

A set may be thought of as a collection of objects. We do not formally define the terms set and element.

The objects contained in a given set are called members or elements of the set.

One method of naming sets is shown below:

$A = \{ \text{Bob, Bill, Tom} \}$

This is read 'A is the set whose members are Bob, Bill and Tom'. Capital letters are usually used to denote sets. The braces  $\{ \}$ , also denote a set. The names of the members of the set are listed, separated by commas, and then enclosed within braces.

In some cases it is impossible to list all the members of a set. For example, it would be impossible to list all the members of the set of numbers greater than 2, but it is possible to use a descriptive phrase. This would be written as:

$B = \{ \text{the numbers greater than 2} \}$

The phrase which we use describes a property which all the elements of a set have in common. This is often referred to as a characteristic property of the set.

An alternative use of the braces notation is illustrated below:

$W = \{ \text{Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday} \}$

Using the set  $W$  above, we can say :

Monday is a member of  $W$

Saturday is a member of  $W$

We abbreviate the phrase 'is a member of' by using the Greek letter epsilon,  $\epsilon$ , to stand for this phrase. Then instead of the above we write :

Monday  $\epsilon$   $W$

Saturday  $\epsilon$   $W$

The slant bar,  $/$ , is often used to negate the meaning on a mathematical symbol. The mathematical symbol  $\notin$  is therefore read, 'is not a member of'. For set  $W$  we can then say :

June  $\notin$   $W$ ,

April  $\notin$   $W$

The symbols denoting the individual members of a set are generally the small letters of our alphabet, such as  $a, b, c, d$ , and so on.

So far we have talked about sets having one or two elements at least. There are, however, some occasions on which we think of a set and then discover that it has no members at all. It is just as natural that a set has no elements as it is to have one, two or more elements. It is certainly mathematically convenient to consider a collection containing no members, as a set, and naturally we call it an empty set. It is denoted by  $\emptyset$ .

#### SET EQUALITY

If  $A$  and  $B$  are names for sets,  $A = B$  means that set  $A$  has the same members as set  $B$ , or that  $A$  and  $B$  are two names for the same set. The order in which the members are listed does not matter. For example,

$$\{a, b, c\} = \{c, a, b\} = \{b, c, a\}$$

## EQUIVALENT SETS

Suppose you had some cups and some saucers. Some one asks, 'Are there more cups than saucers? Would you have to count the objects in each set to answer the question?

All you need to do is place one cup on each saucer until all the members of one of the sets have been used. If there are some cups left over, then there are more saucers than cups. In each case, a cup saucer is paired with one, and only one, saucer, and each saucer is paired with one, and only one, cup as we say the sets are matched one-to-one or that there is a one-to-one correspondence between the sets when there are an equal number of cups and saucers.

The idea of equivalent sets is not the same as that of equal sets. That is two sets are equal if they have the same members. Two equivalent sets may have different members just so long as there exists a one-to-one correspondence between them. Clearly all equivalent sets have the same number of elements as long as they are finite in number. For example :

a, b, c, d is equivalent to r, s, t, u  
but a, b, c, d is not equal to r, s, t, u

## SUBSETS

It is often necessary to think of sets that are part of another set or are sets within a set.

The set of chairs C in a room is a set within the set of all pieces of furniture F in that room. Obviously, every chair in the room is a member of set C and also a member of set F. This and similar examples lead to the idea of a subset.

"Set A is a subset of set B" means that every member of set A is also a member of set B. The symbol of  $\subset$  is used to denote is a subset of a set.

Consider the following sets :

$R = \{a, b, c, \}$      $A = \{a\}$      $B = \{b\}$      $C = \{c\}$      $D = \{a, b\}$   
 $S = \{a, c, \}$      $T = \{b, c\}$      $\emptyset$  . Here A, B, C, D, S, T,  $\emptyset$

are subsets of R. R is a subset of itself. Infact it can

be verified that if A has  $n$  distinct elements it has  $2^n$  subsets.

### UNION OF SETS

We are accustomed to joining sets in our daily activities. For example, when you put some coins in your purse, you are joining two sets of coins - the set of coins in your purse and the set of coins you are about to put in your purse.

Young children will often place objects into sets corresponding to colour and will unite a blue set with a red set and then separate them again without much idea of the number of objects there are. These and many other examples indicate that the idea of combining sets in some way is more elementary than counting. The simple combination of sets we have just mentioned is called a union of the two sets.

### INTERSECTION OF SETS

Suppose a teacher asked a class, "How many of you went to the game last night?" Then several children raised their hands. Those who raised their hands are members of the set of children in the class, and they are also members of the set of all children who went to the game last night.

We can illustrate this situation by using a Venn diagram. Let  $A = \{\text{all children in the class}\}$  and let  $B = \{\text{all children who went to the game last night}\}$ .



Then  $C = \{\text{all children in A, who are also in B}\}$  is called the intersection of A and B and represented by the shaded region in the figure.

It is natural that some sets have no elements in common - such as  $A = \{p, q, r\}$  and  $B = \{x, y, z\}$ , i.e.

$$A \cap B = \emptyset$$

Such sets are called disjoint sets.

## UNIVERSAL SETS

Consider a problem involving sets. In one problem we may discuss cars. It would therefore be necessary to state at the beginning what we are going to consider whether they are two or three or four wheelers and so on. This background set which contains all the sets in the problem as subsets is called the universal set. It is denoted by  $\xi$ .

## COMPLEMENT

So far we have spoken of elements which belong to sets, but as soon as we know  $\xi$  we are able to speak precisely about those elements which do not belong to a set; in other words, the complement of the set with respect to  $\xi$ .

## LINEAR EQUATIONS IN ONE VARIABLE

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Any polynomial equation of degree one is called a linear equation. For example :  $x+y+z+3=0$ ,  $x+y+7=0$ ,  $2x+3=0$ . If an equation has more than one variable it is not always possible to get a unique solution, for example  $x=0$ ,  $y=0$ ,  $z=-3$ , and  $x=z=0$ , and  $y=-3$  are solutions of the first equation listed above. In the case of  $2x+3=0$ ,  $x = \frac{-3}{2}$  is the only solution. We shall consider linear equations of one variable only.

Any equation of the form  $ax+b=0$ ,  $a \neq 0$  is called a linear equation. The solution of this equation depends on the domain of the unknown  $x$ . If we consider  $a$  and  $b$  to be rational numbers then  $x = \frac{-b}{a}$  is the solution of the equation over the set of rationals. If  $x$  can take only natural numbers then the existence of solution depends largely on  $-b/a$ . Consider  $x+1=0$ . It has no solution over the set of Natural numbers and  $x \neq +1$

is the solution over the set of integers. Thus while solving an equation it is necessary to specify the domain of the unknown. Solving an equation is like playing a game. One needs to know the rules of the game before one makes the first move. We shall list some of the rules before we solve any problem on equations.

The solution of an equation is not altered by

- (i) adding any number to both the sides of an equation.
- (ii) subtracting any number from both the sides of an equation
- (iii) Multiplying or dividing by a non-zero constant both the sides of an equation.

Let us consider equations which can be reduced to linear. Consider the general form:

$$\frac{ax+b}{cx+d} = k; \quad x \neq -\frac{d}{c}$$

Here the condition  $x \neq -\frac{d}{c}$  is necessary to make the left hand side of the equation meaningful. Since  $cx+d \neq 0$ , the given equation can be reduced as  $ax+b=k(cx+d)$ .

$$ax+b = ckx+kd$$

$$\text{or} \quad ax - ckx = kd - b$$

$$\text{or} \quad (a-ck)x = kd - b$$

$$\text{or} \quad x = \frac{kd - b}{a - ck}$$

Let us consider an example:

$$\text{Solve; } \frac{5x-7}{3x} = 2, \quad x \neq 0, x \text{ is a rational}$$

Solution:

$$\frac{5x-7}{3x} = 2$$

$$\text{or} \quad 5x - 7 = (3x)(2)$$

$$\text{or} \quad 5x = 6x + 7$$

$$\text{or} \quad 5x - 6x = 7$$

$$\text{or} \quad -x = 7$$

$$\text{or} \quad x = -7$$

Hence  $x = -7$  is the solution of the given equation. Let us check whether the solution is correct. For this we replace  $x$  by  $-7$  in the given equation.

$$\text{Here } \frac{5(-7) - 7}{3(-7) - 7} = 2$$

$$\text{or } = \frac{-35 - 7}{-21 - 7} = 2$$

$$\text{or } = \frac{-42}{-21} = 2$$

which is true. Thus  $x = -7$  is the solution. You may try the following problems.

$$(i) \frac{4x + 3}{2x + 6} = -3, x \neq -3, x \text{ rational}$$

$$(ii) \frac{2-y}{y+7} = \frac{3}{5}, y \neq -7, y \text{ rational}$$

while solving the day-to-day problems where it is required to find an unknown quantity we follow the following steps:

- I. Read the problem carefully and identify the known and unknown quantities.
- II. List the known quantities and denote the unknown as 'x'
- III. Translate the problem into mathematical expressions
- IV. Identify the equal quantities and form the equation
- V. Solve the equation for a possible solution
- VI. Check the solution by replacing the unknown quantity by the solution.

CONTENT ENRICHMENT PROGRAMME IN TEACHING OF  
MATHEMATICS (FOR ELEMENTARY LEVEL I.E. LOWER &  
UPPER PRIMARY) (12.3.1993 TO 16.3.1993).

MR.K.M.SYIEM

DAY - I

(1)

The General objectives of teaching Mathematics are to impart to the children the ability to -

- i) think
- ii) reason
- iii) express oneself logically and systematically
- iv) analyse
- v) pay attention to details
- vi) be accurate

The Number System

The significance of the terms 'Number' and 'Numeral' and their difference.

Number denotes the idea of quantity

Numeral is the symbol to represent the 'Number'

Different languages use different symbols to represent the same quantity e.g. 3(English), ३(Hindi), /۳ (Urdu).

Also, different systems of Numeration use different symbols to represent the same quantity/number :

	Five	Tens	Hundred
Egyptian system	lllll		?
Roman	V	X	C
Hindu-Arabic system	5	10	100

The Hindu-Arabic system of Numeration

Concept of Place Value :- The system uses only ten symbols called 'digits' viz. 0,1,2,3 .....8,9. It uses groups of tens and it is based on the concept of place value. This concept gives it the advantage over the other systems. Because of Place value concept, it is possible to write any number, however large, by using only the ten digits. For example, to write the number 'eighty Eight' :-

- i) Roman system - LXXXVIII
- ii) Hindu Arabic system - 88

### Place Value Chart :

Ten Thousands	Thousand	Hundreds	Tens	Units/Ones
10,000	1,000	100	10	1

Each digit has a face - value and a place - value. Place-value depends on the place that it occupies in the place value chart, whereas face-value of a digit remains the same, regardless of the place that it occupies. For example, in the number 88, the face value of the digit is 8, but the place value of the digit in the Unit's place is  $8 \times 1 = 8$  and in the tens place is  $8 \times 10 = 80$ .

Pattern of writing numbers : Rotation of digits 0 to 9 in cyclic order - everytime we have a no. with 0's only, viz. 9,99,999 etc, to write the next no. we extend one more room in the left of the place - value chart (pvc) and write zero (s) in the existing place (a) and 1 in the new place.

### Natural numbers, whole numbers

Observations : (i) Every no. is 1 more than the no. before (1).

(ii) To get the next number, we just add 1 to the preceeding no.

This fact shows that there is no largest number which can be demonstrated by the possibility of counting of grains of sand in the Sahara desert (say). The act of counting is actually the 'matching' or 'pairing' of a no. (natural no.) with an object.

Pile of sand on one side and pile of numbers on another side and matching or pairing each grain of sand with a no.

Writing of a number in the Expanded form significance.

No largest Number  $\Rightarrow$  pvc can be extended indefinitely towards the left hand side.

## DAY - 2

Decimals : Any place in the PVC is 10 times the next place on its right - thousands is 10 times hundreds, hundreds in 10 times tens and so on. Continuing the argument in this way, we see that the pvc can also be extended towards the right of the units place indefinitely - hence decimals. Thus PVC becomes -

Thousands	Hundreds	Tens	Units	Decimal	Tenths	Hundredths
1,000	100	10	1	point	1/10	1/100

Metric System - Based on the Decimal system - that is, on the above PVC. Three common measurements - length, weight and capacity - the units of which are metre (m), Gram, (g) and litre (l).

Kilo = 1000, Hecto = 100, Deca = 10, Unit (m/g/l),  
 Deci = 1/10, centi = 1/100, milli = 1/1000.

Factors and Multiples : What is factor and what is Multiple?

The factor of 6 are 1, 2, 3, and 6.

6 is a multiple of each 1, 2, 3, and 6

1, 2, 3, 4, 6 and 12 are factors of 12. If we divide 12 by any one of 1, 2, 3, 4, 6 & 12 to get the remainder 0. So, we conclude.

- i) If a number 'b' is a factor of a number 'a', then 'a' divided by 'b' gives the remainder 0.
- ii) 1 is a factor of any number
- iii) Any number is a factor of itself
- iv) If 'a' is a factor of 'b' and 'b' is a factor of 'a', then  $a = b$ .

Prime and composite numbers:

Natural numbers are classified into the following categories;

- i) Numbers having exactly one factor.
- ii) " " " two factors.
- iii) " " " more than two factors.

- a) The no. 1 falls under category (i)
- b) 2, 3, 5, 7, 11, 13 ..... fall under category (ii) -they are called Prime Numbers
- c) 4, 6, 8, 9, 10, 12, 14 ..... fall under category (iii) -they are called Composite Nos.

Definition of Prime Numbers, Composite Numbers:

Twin Primes : Prime nos. having only one Composite no. between them e.g. 3, 5; 5, 7; 11, 13; 17, 19; 29, 31; 41, 43; 59, 61; 71, 73 are twin primes below 100.

Relative Prime numbers : If their HCF is 1.

Multiplication of Integers : Rule of signs ..

We know,  $3 \times 5 = 5 + 5 + 5 = 15$

$$\begin{aligned} \therefore 4 \times (-5) &= (-5) + (-5) + (-5) + (-5) \\ &= -20 \\ &= -(4 \times 5) \end{aligned}$$

Consider  $(-4) \times 3$  : We have

$$\begin{aligned} (-4) \times 3 &= 3 \times (-4), \text{ (why?)} \\ &= (-4) + (-4) + (-4) = -12 \\ &= -(3 \times 4). \end{aligned}$$

$$\therefore (+ve) \times (-ve) = (-ve) \times (+ve) = -ve \dots\dots\dots(1)$$

Alternatively, (through pattern),

$$\begin{array}{lcl} 3 \times 3 = 9 & & \\ 2 \times 2 = 6(9 - 3) & & \\ 3 \times 1 = 3(6 - 3) & & \\ 3 \times 0 = 0(3 - 3) & & \\ 3 \times (-1) = -3(0 - 3) & & \end{array} \left. \begin{array}{l} \} \\ \} \\ \} \\ \} \end{array} \right\} \begin{array}{l} \text{decreasing successively} \\ \text{by 3} \end{array}$$

Multiplication of like signs ..

Consider  $(-ve) \times (-ve)$

Let us find out  $(-3) \times (-2)$

Observe the following :  $(-3) \times 4 = -12$   $\left[ (-3) + (-3) + (-3) + (-3) \right]$

$$(-3) \times 3 = -9 \quad \text{i.e. one } (-3) \text{ less.}$$

$$(-3) \times 2 = -6$$

$$(-3) \times 1 = -3$$

$$(-3) \times 0 = 0$$

$$(-3) \times (-1) = ?$$

We observe that as the 2nd integer decreases by 1, the product increases by 3. Therefore, the product  $(-3) \times (-1)$  should be 3 more than 0 i.e. 3.

Similarly,  $(-3) \times (-2)$  should be 3 more than 3 i.e. 6., so on. Thus  $(-3) \times (-2) = 6 = +(3 \times 2)$

The Distributive property of multiplication over addition also helps in arriving at the same result.

We have,  $(-3) \times 2 + (-3) \times (-2) = -3 \times 0 = 0.$

$$\text{or, } (-3) \times 2 + (-3) \times (-2) = 0$$

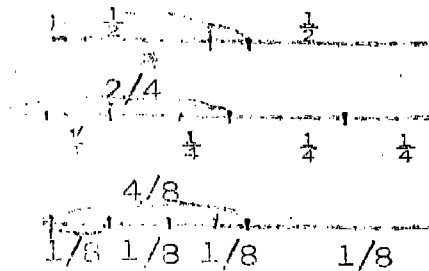
$$\text{or, } (-6) + (-3) \times (-2) = 0$$

$$\text{or, } (-6) + ? = 0$$

Now, we want to find out - What is to be added to  $(-6)$  so that the sum may be 0? The answer is  $+6$

$$\text{Hence } (-3) \times (-2) = 6 = +(3 \times 2)$$

Equivalent Fractions:



$\frac{1}{2}$ ,  $\frac{2}{4}$ ,  $\frac{4}{8}$  represent the same quantity ( in this case the same length). They are known as equivalent factors.

(i) If both numerator and denominator of fraction are multiplied or divided by the same quantity (number), the value of the fraction is not changed, it is simply changed into an equivalent fraction.

Consider  $\frac{2}{3}$ ,  $\frac{3}{5}$  which one is greater ?

$$\left. \begin{aligned} \frac{2}{3} &= \frac{2 \times 5}{3 \times 5} = \frac{10}{15} \\ \frac{3}{5} &= \frac{3 \times 3}{5 \times 3} = \frac{9}{15} \end{aligned} \right\} \therefore \frac{2}{3} > \frac{3}{5}$$

By expressing as equivalent fractions with the same denominator/numerator we can easily tell which is greater.

Consider,  $\frac{2}{3} + \frac{3}{5}$

Converting them into equivalent fractions with the same denominator, we have

$$\frac{2}{3} + \frac{3}{5} = \frac{2 \times 5}{3 \times 5} + \frac{3 \times 3}{5 \times 3}$$

$$= \frac{10}{15} + \frac{9}{15}$$

$$= \frac{10+9}{15} = \frac{19}{15}$$

Usual Process :

$$2/3 \pm 2/5 = \frac{2 \times (5)}{15} \pm \frac{2 \times (3)}{15} = \frac{10 \pm 6}{15} = \frac{16}{15} \text{ or, } \frac{1}{15}$$

That is, when we add or subtract fractions, we simply express them as equivalent fractions with the same denominator (LCM of denominators) and then add or subtract the numerators.

HCF by continued division method : Why ?

HCF of 18,30 is the same as finding the length of the largest rod that can measure two rods of 18 cm and 30 cm. an exact number of times.

A 30 cm B  
C 18 cm D

By cutting a larger rod into pieces each equal to the length of the smaller rod, we get one piece and a length 12 cm is left as below :

A 18cm E 12 cm B  
C 18 cm D

We divide 30 by 18 as,

$$\begin{array}{r} 18 \overline{) 30} ( 1 \\ \underline{18} \\ 12 \end{array}$$

The rod which will measure CD an exact number of times will obviously measure the 18cm piece of AB an exact of times.

The required rod must measure the remaining portion of the larger rod i.e., 18cm (smaller rod) an exact no. of times.

Next, C 12cm D  
12cm 6cm  
E 12 cm B

Next we divided 18(the 1st divisor) by 12(the remainder)

$$\begin{array}{r} 12 \overline{) 18} ( 1 \\ \underline{12} \\ 6 \end{array}$$

By the same reasoning, we get that the required rod must measure the remaining portion/of CD i.e., 6cm and 12 cm and exact number of times.

We now cut the 12cm piece into pieces each equal to the length of 6cm. We get exactly two such pieces and no portion left over.

6cm 6cm  
6cm

Next divider 12(the 2nd divisor)  
by 6(the 2nd remainder)

$$\begin{array}{r} 6 \overline{) 12} \quad ( 2 \\ \underline{12} \\ 0 \end{array}$$

Obviously, the required largest rod that will measure both the given rods an exact number times must be of length 6cm. That is, the HCF of 18,10 is 6. Hence the process continues.

Division by zero is not defined.

### Base 2 or Binary system of Numeration :

Present system - Decimal or base - 10 system  
because there are ten symbols  
or digits and we use Groups of  
Ten.

In Base - 2 - Only two symbols viz. 0,1.  
And Place value is in Groups of 2.

### Place Value Chart.

Thirty twos	Sixteen	Eights	Fours	Twos	Units
32	16	8	4	2	1
4	3	2	1	0	
2	2	2	2	2	2

We write 2 after below a no. to indicate that is in base - 2 system.

To convert a numeral from base 2 to base -10.

Express the numeral in expanded form.

Example : Convert 101101 as a numeral in base - 10 system.

Writing the numeral in place value chart:

5	4	3	2	1	0
2	2	2	2	2	2
1	0	1	1	0	1

$$\begin{aligned} & \text{---32---} \\ \text{(i) } 101101_2 &= 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 \\ & \quad + 1 \times 2^0 = 1 \times 32 + 0 + 1 \times 8 + 1 \times 4 + 0 + 1 \times 1 \\ & \quad = 32 + 8 + 4 + 1 = 45 \end{aligned}$$

$$\text{(ii) } 111011_2 = 32 + 16 + 8 + 0 + 2 + 1 = 59$$

To convert a numeral from base -10 to base -2

We divide successively by 2 until the last quotient is 0.

Example: Change 59 to a numeral in base -2.

Recall the positional values in base -2, viz., Units, Twos, Fours, Eights, etc., We divide 59 by 2. We get a quotient of 29 and remainder 1. This means that there are 29 twos in 59 and 1 unit. We write 1 in Units place. We next distribute 29 twos in the other position values of base -2.

29 divided by 2 gives 14 as quotient with remainder 1. Thus, 29 twos contain 14 fours and 1 two. We write 1 in twos' place and so on.

$$2/59$$

$$2/29, 1 \text{ Units}$$

$$2/14, 1$$

$$2/7, 1$$

$$2/3, 1$$

$$2/1, 1$$

$$0, 1$$

$$\text{Thus } 59 = 111022_2$$

Exercise:

1) Convert to base -10  
11, 110, 111, 101, 1101, 11001, etc.

2) Convert to base -2  
4, 12, 23, 14, 68, 33, 29, .....

Division of fractions :

$$\begin{aligned} 2/3 \div 3/4 &= \frac{2/3}{3/4} = \frac{4/3}{4/3} \quad (\text{to make } D.F. = 1) \\ &= \frac{2/3 \times 4/3}{1} = \frac{2}{3} \times \frac{4}{3} = \frac{8}{9} \end{aligned}$$

$$\text{Hence } 2/3 \div 3/4 = 2/3 \times 4/3 = 8/9.$$

THE HISTORY OF MATHEMATICS  
-ITS TEACHINGS AND EFFECT

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Synopsis

The History of Mathematics is rightly said to be the history of our civilization, specially as regards to the development of science and technology. Through the teachings of history of mathematics, it is easy to know the nature and role of mathematics, its growth and spread and its involvement with other branches of knowledge, not only from its outward manifestations but from its internal texture also. Infact, the abstractness of mathematics have always been a dominant force through its practical use in the rise of scientific and technological world. So such thrilling and useful history of mathematics if told to the learners in the classroom situations will inspire them (i) to avoid avoidance in mathematics, (ii) to get rid of fear psychosis tamed in the name of mathematics and also (iii) to inculcate love for mathematics.

The history of mathematics has several values, some of which can be stated as follows:

- (a) Through the history of mathematics it is possible to present the teaching unit as a dynamic and progressive subject creating human interest.
- (b) Through the discussion of history it is possible to exhibit the utility of the subject, removing the faulty notion that mathematics is a dry subject far away from life.
- (c) The romance of discovering the topics can help the teachers to remove the monotony of the students in the classroom teaching.
- (d) The knowledge of history of mathematics of a teacher can command respect from the students adding something to the personality of the teacher.
- (e) Many terms, concepts and conventions can properly be understood or explained through the discussion of historical development of the topic.

(f) The discussion of history of mathematics helps to grade the subject and to establish correlation of mathematics with other subjects which is very necessary to show the importance of mathematics.

(g) The logical and systematic order of the subject matter can be established through the discussion of the history.

(h) The history of mathematics reveals a very important fact that mathematics an essential subject for the growth of our civilization, specially the science and technology achieved today is nothing but a man-made science. This spirit if properly exhibited will encourage the learners more towards learning of the subject and inculcate the feeling that he/she too can contribute something towards the growth of mathematics.

For example, the historical review of the development of the numeration .. system, metric .. system of weights and measures, logarithms, computers etc will always encourage the learners to love mathematics. Similarly, the contribution of the ancient Hindu Mathematicians to the development of mathematics will inspire the learners. The contributions specially by Brahmagupta, Aryabhatta, Bhaskara and Ramanujan are relevant to school mathematics.

Thus we can aptly remark that 'No subject loses more than mathematics by any attempt to dissociate it from its history'. Then discussion on above followed.

## IMPORTANCE OF TEACHING MATHEMATICS AT ELEMENTARY STAGE

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Nature : Mathematics is talked in terms of 'Science of Quantity and Space'. Infact it is a systematised, organised and exact branch of Science'. Kant even explained it, as the gate way to all Sciences'. Its study puts a man on coherent thinking and logical reasoning. Mathematics helps a child to apply his mind to receive, collate and interpret the ideas and themes gained from various sources of life. It drills the brain to conceive exact nature of thinking and apply the reasoning. Its studies provide comprehensive knowledge and disciplined mind.

D. Hilbert says, 'Mathematics is always full of life as there is always an abundance of problems'. Weyl explains, 'all great battles of Mathematics and Physics are fought on fields of characteristic Zero'. According to Maxwell, 'Mathematics is the art of saying the same thing in many different ways'. 'Mathematics is engaged, infact, in the profound study of art and the expression of beauty' says Mr. Shaw.

Mathematics has many explanations to offer. It is full of life, challenges, problems and encourages the readers to confront them with courage, conviction and patience. In numeration it battles out at Zero unlike physics. In the field of aesthetics, it is known as an art, recreation. It offers a beautiful explanation of natural phenomena and phenomenal description is done by performing mathematical activities.

### Concept . Meaning :

The children are in the formative period at this stage. They are in the process of development - physical, mental, emotional and social. They observe, makes hypotheses, experiments and draw inference. This creative brain always needs explanation to facts and visions observed outside. The fertile brain of the children keeps

its cells open to receive as much knowledge as it is possible at this juncture. It will be only mathematics which can control, expand and explain the mind of the children as well as their behaviours. The continuous doing of mathematical sums leads to acquire numerical ability, to cultivate a right type of habit, attitude, intellect and aptitude. The continuous practice of mathematics is very much needed to develop a child into the full fledged man because it shapes the activities and behaviour of growing adult at school and college stage.

Mathematical activities train up the mind into a perfect disciplinarian. Schultze subscribes these views and thus says 'Mathematics is primarily taught as an account of mental training it affords and only secondarily on account the knowledge of facts it imparts'. Mental training is highly essential for enabling to function like a human computer with reason. Locke understands, 'Mathematics as a way to settle in the mind a habit of reasoning'. Here a child can apply his knowledge and mind to the fullest achievement whatsoever is.

The pertinent question remained to be answered. Is the teaching of mathematics all that necessary at every stage after putting much efforts, time and money? Should it be made compulsory and thus included in all the curriculum for teachers and students to involve to such an extent? These questions need explanation to the utmost satisfaction of the teachers, students and guardians.

The teacher first have to get convinced with the needs of mathematics teaching and then he has to convince the concerned parties. He should develop a keen interest, create a right type of teaching and learning situation and motivate the children through his skillful techniques of teaching. The discussion leads us to gather the fact that mathematics teaching has much importance and values. Let us discuss some of its values :

1. Practical Values : Since the birth of the child mathematical processes are involved in some ways or another. His physical, mental, emotional and social development take place on the principles of counting,

calculation and evaluation. The steps the child puts to move forward, learn mother tongue bit by bit, express his feelings inform of 'cry', socialise with family members and others; all such activities are performed mathematically. This is the practical or utilitarian value of mathematics which are being felt right from beginning. Its utility increases more with the growth of child into man or till the end.

The educated man understands the value of mathematics to a great extent. Even illiterates know counting and verbal numeration for their benefit. Sometimes it happens that some of labours do not know calculation and thus cheated by their masters. However generally it is observed that the mathematics skills are available with almost all the people whether educated or not. All sorts of people save whatever possible out of their earning for future uses. Their children ask fifty paise, one rupees or as much they need to purchase the toys, sweets, etc. of their choice. They keep the verbal accounts of the expenditure incurred to explain their elders. Thus earning and spending which is a part of mathematics has become an integrated instinct of human beings.

The four fundamental rules of mathematics are taught at the elementary level. The children learn systematically addition, subtraction, multiplication, division, counting, weighing, measuring etc. which knowledge they are applying in common life. Automatically they come to know the importance and value of mathematics which make them sure about what they intend to do.

The children always like to play, do some creative work. On doing such things they practice mathematics knowledge. The children observe flying of aeroplane in the sky, functioning of natural elements as per schedule, working of T.V; Radio, Computer which are useful and pertinent events in our regular life. Such activities are occurring mathematically. Rightly A.G.Kemeny observed, 'whether man's travels carry him into space or into theoretical science, his passport must be stamped with the mathematicians seal of approval'.

The children at elementary level understand the value of money as without it nothing can be purchased for their uses. They learn loss and profit, counting, percentage etc. and thus plan about their future to become adolescent, adult, citizen of the society, state and the nation. They become economical in dealings which have linkage with the social and state development. Napoleon rightly said, 'The progress and the improvement of mathematics are linked to the prosperity of the State'. There is no denying the fact that it is mathematics which knowledge puts a man in inventive position. Certainly mathematics has practising value in man's life.

## 2. Disciplinary Value

The mathematics study enables a child to adapt the situation in which he was put in. It refreshes the kind and sets reading instincts in a right way. Locke feels 'Mathematics is a way to settle in the mind a habit of reasoning'. Doing mathematical sums drills the mind and puts it on exercises. It inculcates right and positive attitude in the learners, so that they can think, reason and pass on the judgement properly. The patience gained through mathematical calculation keeps the child in check, control instincts to become disciplined. Infact a mathematician is true disciplinarian as he is a man of few words. The study provides him power to decide correct line of action.

The constant exercises of mind develop power to think reason, be exact, precise, gain probing capacity, to reach an exact solution, to verify and cross-check the results and to be accurate in dealing with the problems. Such activities always engage the mind so the man does not go astray rather he becomes adroit and disciplined. The involvement of children at this stage in numeration, doing verbal and non verbal ability tests, counting, doing percentage, loss and profit etc discipline the mind and the man to the par-excellence. The man of mathematics would never be crude but soft hearted, kind and a good citizen of the society. So mathematics teaching has emense disciplinary value.

### 3. Cultural Value

The laws and postulates of all sciences are based on the mathematical concepts. Physics and Astronomy being exact science culminate all its usefulness in mathematical calculations. The computer science, the modern most desired field of study represents the totality of mathematical behaviours and its study creates a culture, the habit, way of living and performing and systematises the role and actions. It helps in understanding the social evolution and its functioning. The social set up and stratification are based on mathematical norms.

F.G. Smith, so accepts, 'It is felt that we have reached a point in almost every sphere of human activity and quasi achievement where all majority problems of business, industry and government, can be formulated and presented as systems of mathematical logic'. All works of life are so augmented with mathematical compulsions and considerations that the children at elementary stage need to be brought to such considerations. They need to be civilised, advanced and organised systematically. The successful launching of INSAT -2B from French Guiana base on 23.7.93 has influenced the minds of the entire nation specially the children. The atmosphere is surcharged. The children so need to be properly guided to create a class, a homogeneous culture to enable every body live free from tension.

The history of heritage and culture has been preserved in chronological order based on mathematical recycling so its teaching at all level is immensely useful. The aesthetic senses hidden in cultural arts, music, painting etc with all postulates represent the central theme of mathematics teaching for which the children are very fond of. It is mathematics which cultural value and importance is worthy to prescribe. Mr. J.V.A. Young rightly remarked, 'were its backbone (Mathematics) removed, the whole of material civilization would inevitably collapse. Mathematics develops cultured citizens who can deliver good to the society'.

#### 4. Scientific Value:

The Society is ever dynamic and so its courses are changing rapidly on the basis of the requirements. There is a wide expectations that the children at elementary level must be sounded with scientific understanding which they can utilise in society rebuilding.

As the children have got probing mind right from their birth they act enquisitively <sup>and</sup> scientifically. The scientific courses follow mathematical calculations. The whole day children's activities are guided strictly by mathematical processes which are very much scientific in approach.

The laws of physics, chemistry etc are derived mathematically. The computer obeys principles of mathematics. The space satellites, soynuz, appolo are designed, developed and launched in the space to reach various distinations and purposes which involve a lot of permutations and combinations. The cure of diseases by medicines, surgery, flying of aeroplanes, running of the trains with high speed, functioning of T.V./Radio are creating curiosity in the children to know deeper. If such realisations are prevailing, certainly interest of learning of mathematics scientifically will emerged.

Mathematics has created a vast area of its own. Emergence of new values, trends and parting ways from many old traditions are taking place due to changing attitude of man after judging its utility after careful considerations. The man is becoming economical, calculative, precise- what does it mean?. It means all the dogmas, culture, civilization now stand for scrutiny due to gain of scientific and mathematical knowledge. The need of the hour is to develop technology and to transmit it to all fields of human life for maximum gain. These are well-planned and calculated steps towards boosting the social economy. So mathematics has scientific values which should be taught to the children at elementary stage.

#### 5. Other Related Values:

The children have abundance of energy which

needs to be channelised. They are clean slate, clean hearted. They need to be inculcated right type of habit, attitude, values, sense of proportion etc. in a systematic way without hurting their sentiments. The children should be economic, less expensive and value the time which never comes again. The postulates of mathematics should be explained in a positive way to be valued by them.

The proper study of mathematics creates artistic frame of mind. It leads to acquire the power of concentration, live simple and economic life, utilise the time and opportunity properly, do quality work, become self dependent, acquire the power of discovery and explanation. In plain world we can conclude that teaching of mathematics leads to comprehend a sense of integrated value by the children at elementary stage. A mathematically trained child becomes an original in dealings.

#### 6. Educational Values:

Everybody knows the minimum uses of mathematics. But in modern life when science is making fast progress and becoming a house hold practice, the knowledge gained in a haphazard way will not work. It has to be taught regularly and systematically. The students need interest to study the subjects, so education is to create an interest and fondness in them. Mathematics has become an integrated part of teaching right from class I therefore, the method of teaching tools and techniques etc have to be renovated. Infact teaching of mathematics have limitless value which can be substantiated through educational practices.

#### Function, Aims & Objective

The subject taught at all levels in formal setting have certain aims and objectives to be achieved. Mathematics teaching is not an exception as it has got much clout with human life. The aims and objectives sometimes get synchronise with the function of mathematics. Inview of this we shall discuss both simultaneously.

The chief aims of teaching mathematics are to increase knowledge and skills, inculcate intellectual habits (pursuits) and power, cultivate desirable attitude and ideals, foster a habit to concentrate, create power to think and reason, stimulate the children to be creative, artistic, fair, calculative and timely. The objectives differ from topics to topics and thus are tried to be achieved during classroom teaching. The objectives of teaching geometry, algebra, arithmetic so will certainly vary. The achievement of objectives helps the teacher and learners to improve their educational activities as per the demand of the students and the guardians.

In fact the functions of mathematics become the aims and objective while doing its in practice. The following are the chief functions of mathematics at elementary level.

1. to develop interest to study or to learn mathematics
2. to develop power of solving problems in daily life
3. to discipline the mind, behaviour and activities of the child.
4. to prepare the child to acquire the habit to become economical, purposeful, productive, professional, scientific, creative and constructive.
5. to prepare the child to take up higher mathematics in further classes.
6. to develop a frame of mind to explore, investigate as 'mathematics is the indispensable instrument of all physical researches' -Berthelot.
7. to cultivate a rational and responsible man through mathematical processes. In fact "in mathematics we find the primitive source of rationality and to mathematics, must the biologists resort for means to carry on their researches" Comte. A.
8. to develop a right scientific attitude, aptitude and the will to labour hard for certain

gains. 'It creates the right type of intellectual climate in which science and technology can flourish'

The elementary level children need the handling of their affairs with cool mind, patience, love and care. They need to be properly directed so that they can achieve the maximum after exploiting the opportunity at hand. Here is a place where mathematician should discharge their duty as a friend, philosopher and guide. The utility of teaching modern mathematics has to be explained in right earnest at this level.

Mathematics is omnipresent. Whatever one is doing, functioning, thinking it follows mathematical processes. It means every work is preceded and acceded with mathematical calculations.

The professionals skilled or unskilled - such as carpenter, blacksmith, tailors, shopkeepers - big or small all use mathematics in their profession. The daily wage calculates his wages. The common people do the marketing and keep the accounts involved. In view of the discussion, it is abundantly clear that mathematics is very important and thus useful in day to day life.

#### Mathematics Curriculum At Elementary Level

The children need integrated development on intellectual front to face the challenges in life. They should know the importance of mathematics teaching. So the teaching of mathematics has been made compulsory in school. The school curriculum up to Class X has been designed in integrated form; mathematics being one of the compulsory subjects.

The children who are involved in science and mathematics learning from beginning certainly will get a systematised brain and disciplined kind of approach to deal with the problems. As per the New Policy on Education (NPE 1986), CERT has published the mathematics books from Class I to XII which are very good and worthy to read.

These books contain modern treatment of mathematical topics to be taught.

Now a days emphasis is given on teaching application side of mathematics. As this is generally known as space age where most of the developed and developing countries do researches after launching their space vehicle or satellites, the study of mathematics becomes more pertinent. The knowledge will enable the children to understand the constitution and functions of such scientific equipments. With the slipping out of the time the importance of mathematics study is increasing. So the teachers have to be oriented time and again to refresh and up date their knowledge.

## TRAINING MATERIAL ON SIMILAR TRIANGLES

(Prepared by Ishwar Chandra )  
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### Introduction

The concept of same shape and different sizes is very much visible in nature. Human beings, leaves of a particular tree, animals of a particular type like cows, lions, etc. and many other things are found having the same shape, of course different sizes. This helps us in naming that particular class of creatures or things. In practical day-to-day life also we find shirts, coats, pants, shoes, underwears, etc. having same shape and different sizes. Similarly, glasses, mugs, cups, plates, screws, needles, cycles, knives and many other items are available having same shapes and different sizes. When we see such objects, we think of their common characteristics/properties. For that purpose, in Geometry, we study figures having same shape. Figures that have the same shape are called 'similar figures'. The basics of such figures are similar rectilinear figures in a plane and the basics of similar rectilinear figures are similar triangles.

The concept of 'similarity' is one of the most important concepts of Geometry as it helps in measuring heights and distances which otherwise could not be measured. A great service done by the science of geometry to humanity is the use of indirect measurement on the basis of the principles of similar triangles.

Thales (about 600 B.C.) is considered to be the originator of this concept. He found the height of a pyramid in Egypt on the basis of length of its shadow by using properties of similar triangles.

### Learning Outcomes

After the study of the chapter, the child must be able to :

1. Use the concept of ratio proportion and properties of proportion in unfamiliar situations.

2. Define similarity of two polygons, specially triangles.
3. Distinguish between 'similarity' and 'congruence'
4. State and prove basic proportionality theorem and its converse.
5. State and prove characteristic properties of similar triangles.
6. State and prove Pythagoras Theorem and its converse
7. Apply the concept of similarity of triangles and Pythagoras Theorem to determine unknown heights and distances.

#### Previous knowledge required

1. The concept of ratio
2. The concept of proportion, properties of proportion and proportional segments.
3. Area of a triangle

Note : The teacher must give a pretest on the previous knowledge required.

#### Content Analysis and Explanation

For the similarity of two rectilinear figures,

- i) corresponding angles must be equal and
- ii) corresponding sides must be proportional

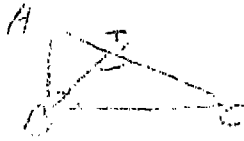
The word 'and' is important since both the conditions are necessary. If only one condition holds, the figures may not be similar as illustrated in the textbook. Triangle is a special type of rectilinear figures. In case of triangles if one of the above conditions holds, the other holds automatically. Thus, either of these becomes the definition of the similarity of two triangles.

Rest proceed as content explained in the textbook.

#### Common Mistakes/Misconceptions

1. While expressing similarity of two triangles, the children do not follow the order pattern of corresponding vertices. For example,  $\triangle ABC \sim \triangle PQR$  is not the same as  $\triangle ABC \sim \triangle QRP$ .

2. Children find it difficult to locate or determine corresponding vertices and sides of two similar triangles in certain circumstances like



- i)  $\triangle ABC \sim \triangle ADB$   
 ii)  $\triangle ADB \sim \triangle BDC$

### Teaching Points/Points of Emphasis

1. In general, two rectilinear figures may not be similar even when their corresponding angles are equal. The triangles are special type of figures in which if corresponding angles are equal, then they are similar.
2. In general, two rectilinear figures may not be similar even then their corresponding sides are proportional. The triangles are special type of figures in which if corresponding sides are proportional, then they are similar.
3. Any two squares, equilateral triangles, circles or regular polygons having equal number of sides are similar.
4. Two congruent figures are also similar but not conversely.
5. While expressing similarity of two triangles in symbolic form, the corresponding vertices must be written in the same order. For example, with reference to the above figure, each of  $\triangle ABC \sim \triangle ADB$ ,  $\triangle BAC \sim \triangle DAB$  or  $\triangle CBA \sim \triangle BDA$  is correct. However,  $\triangle ABC \sim \triangle ABD$  or  $\triangle ABC \sim \triangle BAD$  is not correct.
6. Corresponding sides of similar triangles must be found by locating sides opposite to corresponding angles (vertices).
7. The myth that 'Similar Triangles' is a difficult chapter should be removed by proceeding systematically as above.

### Learning Activities

1. Children may be asked to collect similar objects like marbles, leaves of a tree, etc.
2. Different pupils may be asked to divide various

line segments in ratios like 2 : 3, 3 : 4, 4 : 5, 3 : 7, etc.

3. Let students draw figures similar to given figures. For example a triangle similar to a given triangle having its side double that of the given triangle, a polygon similar to a given polygon having the side of each side as  $\frac{2}{3}$  of the corresponding side of the original polygon.
4. Let pupils draw two similar triangles on a graph paper and determine their areas by counting unit squares enclosed. Let them find the ratio of their areas. Is the ratio nearly the same as the ratio of the squares of corresponding sides?
5. Children may be asked to draw a right triangle, say ABC right angled at B. Let them draw squares with AB, BC and AC as sides. By counting the unit squares enclosed, let them determine the areas of these squares. Is the sum of the areas of the first two squares equal to the area of the third square?

#### Enrichment/Recreational Material

Introduction to trigonometric ratios and problems related to heights and distances may be discussed.

#### Evaluation

##### Blueprint for the Test

Content/Objective	Knowledge	Understanding	Skill	Application	Total
Concept of similarity	2				2
Basic proportionality theorem	2		2	2	6
Characteristic properties of similar triangles	4	10		4	18
Pythagoras Theorem	2	6		2	10
Application of similarity in practical situations		4			4
Total	10	20	2	8	40

Note : Figures in cells indicate marks

## UNIT TESTS

(By Ishwar Chandra )  
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A unit test is not just a random collection of questions. To be an effective instrument of evaluating academic achievement, it has to be structured according to a pattern decided in advance. The following steps are necessary to be taken up for setting a good unit test in mathematics.

### 1. Preparation of a Design

The design of a unit test lays down the chief dimensions of the unit test. In respect of the design, the following points are to be considered.

#### i) Weightage to objectives

This means the selection of objectives to be tested and allotting marks to each in view of its importance. This is a fact that all instructional objectives can not be tested through written examinations. Here, we have selected 'knowledge', 'understanding', 'application' and 'skill' to be tested through written questions and left other objectives like 'interest in mathematics', 'developing proper attitude', 'acquiring personality traits' for the teacher to test through observations.

The student is supposed to have the knowledge of mathematical concepts if he

- a) recognises terms, symbols, facts, etc.
- b) recalls terms, symbols, facts, etc.
- c) uses formulae/rules directly
- d) reproduces a mathematical process

The student is supposed to have developed understanding of mathematical concepts if he

- a) detects errors in formulae, definitions, processes, etc.
- b) corrects errors in formulae, definitions, processes etc.
- c) discriminates between mathematical concepts
- d) gives his own illustrations of mathematical concepts.
- e) translates verbal statements into symbolic form and vice-versa.

- f) explains concepts in his own words.
- g) verifies mathematical results.

The student is supposed to have developed the ability to apply his knowledge and understanding to unfamiliar situations if he

- a) analyses the data into parts
- b) Judges the sufficiency and relevancy of the data
- c) judges the consistency of statements
- d) establishes relationships among the data
- e) suggests alternative methods for solving problems
- f) selects the most appropriate method or line of attack
- g) points out fallacies
- h) draws conclusions or inferences (i.e., reasons deductively).
- i) generalizes (i.e., inductively)
- j) estimates results

The student is supposed to have developed skill in computation, handling mathematical instruments and drawing mathematical figures if he

- a) does oral calculations correctly and quickly
- b) does written computations correctly, quickly, systematically and legibly
- c) handles mathematical instruments with care and speed
- d) measures accurately and speedily
- e) draws free hand figures fairly, accurately and speedily
- f) draws figures to specifications and scale
- g) draws figures and graphs neatly and speedily
- h) interprets tables, charts and graphs
- i) estimates magnitudes

## ii) Weightage to Different Areas of Content

This relates to the analysis of the syllabus and the delimitation of the scope of each topic and then the allotment of marks to each topic and then the allotment of marks to each topic for the purpose of framing questions.

## iii) Weightage to Different Forms of Questions

Many times to test a particular ability a particular form of question is more suitable. For example to

discriminate between closely related concepts 'multiple choice type' question is more suitable. Various forms of questions are based upon free response and fixed response. Essay type and short answer type come in the category of free response and multiple choice type, true-false type, matching type and mostly very short answer type in the category of fixed response. In view of the above, we may like to choose different forms of questions for inclusion in the unit test. Having made this decision, the marks to be allotted to each form have also to be decided.

#### iv) Weightage to Difficulty Level of Questions

A question is said to be easy if it can be solved correctly by more than 70% of the total number of students. If it can be solved correctly by 30% to 70% of the total number of students, it is called average and if less than 30% of the total number of students can solve it correctly, it is called difficult. Decision has also to be taken regarding the distribution of difficulty level of questions in the unit test. In the unit tests that follows we have taken 50% questions of average difficulty level, 30% questions of difficult type and 20% questions of easy type.

#### v) Scheme of Options and Sections

In view of remedial measures to be undertaken by the teacher, no choices (options) have been provided in the unit test. All questions are compulsory.

#### Vi) Decision about the Time and Marks

Keeping in view the load of content and instructional objectives a decision about the time to be allotted and the total marks for the test is to be taken. In the unit tests total marks and the time are usually mentioned on the top of each of the tests.

### 2. Preparation of the Blue Print for the Unit Test

A blue print relates the details of the design in concrete terms. It is a three dimensional chart giving the placement of questions in respect of

- i) the objective tested by each

- ii) the content area covered by each and
- iii) the form of question which is most suitable for testing (i) and (ii) above.

In addition to the above three dimensions, the blue print may also indicate

- i) The score for each question individually and
- ii) The scheme of options and sections to be adopted in framing the questions.

### 3. Preparation of Questions Based on the Blue Print

By taking each cell of the blue print, individual questions are to be framed to satisfy the different dimensions of the respective cells. The framing of questions based on the respective cells. The framing of questions based on the blue print should contribute the knowledge of objectives and their specifications, a mastery over subject matter and the skill in framing different forms of questions. While writing questions for the unit test it may also be kept in mind that the question is at the desired level of difficulty and the language is well within the comprehension of the students. It must also clearly indicate the scope of the answer without any ambiguity.

### 4. Assembling and Arranging Questions

There are various ways of arranging the questions depending upon the form, content, objectives or difficulty level. Each method has its good and bad points. However, the most popular method is to arrange them according to their forms. The first few questions should invariably be of low difficulty level so that the students do not get a psychological set-back right in the beginning.

### 5. Instructions to Examinees

The test paper will be complete only if it contains the clear directions for students. General instructions for the test must be given in the beginning of the paper and specific instructions related to a particular question or section may be given with the respective question or section.

# LESSON PLAN 1

SUBJECT .. ALGEBRA

CLASS .. IX

LESSON UNIT .. POLYNOMIALS

TEACHER'S NAME .. ISHWAR NATH SING

## (1) General Aims :-

- (i) To develop reasoning, thinking and imagination power of the student.
- (ii) To develop interest in the study of Algebraic expression and to acquaint them with the rules and principles of polynomials.

## (2) Specific aims :-

- (i) To enable them to consider a special type of algebraic expression involving only one variable.
- (ii) To enable them to apply the formula to relevant exercises.

## (3) Teaching Aids :-

Colour Chalk, Pencil, Duston etc.

## (4) Previous Knowledge :-

The students are expected to know the term, known and unknown quantity, constant.

## (5) Preparation :- To test the previous knowledge of the pupils, the following questions will be asked:

- (i) What is degree?
- (ii) What is natural number?
- (iii) Which term is called constant?
- (iv) Do you know about monomial, binomial and trinomial?

## (6) Announcement of Lesson :-

The teacher will tell that he would be teaching polynomials.

## (7) Presentation :-

Teaching points	Objectives and method	Matter/Black Board/Teacher work	Discussion	Remark
(1) Explanation of polynomials	To give idea about polynomial, the teacher will ask a question; (i) what is rational number?	Def-A function $P(x)$ of the form $P(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$ where $a_0, a_1, a_2, \dots, a_n$ are constant and $n \in \mathbb{N}$ is called polynomials.	N for natural number i.e. 1, 2, 3, 4, 5, ....	
		$a_0, a_1, a_2, \dots, a_n$ are also called coefficients.		

of the polynomials.

Example:-

(i)  $5x^2 - 6x + 3$  is a polynomial.

(ii)  $\frac{4}{3}x^3 - 2x - 1$  is polynomial over real.

(iii)  $3x^2 + 2x - 3$  is a real polynomial.

(iv)  $x + 1$  is not polynomial because it is getting inverse degree.

(ii) Discussion on monomial, binomial and trinomial.

(i) Monomial:- A polynomial having one term is called monomial. Monomial for one. For example  $2, 2x, 7x^2$  etc,

In number (i) the coefficient of  $x^2 = 3$   
 $x^2 = 7$  etc.

(ii) Binomial:- A polynomial having two terms is called binomial. 'bi' stands for two. For example  $4 - 2x, 3u^4 + 2u, yx^2 - y$  etc.

(iii) A polynomial having three terms is called trinomial. For example  $x^3 + 2x + 2, t^2 - 3t + 3$  etc.

(iv) A polynomial having all coefficients as zero is called zero polynomials. For example  $0x^3 - 0x + 0, 0t^2 + 0t + 0$  etc.

(8) Recapitulation :-

(i) Which of the following functions are polynomials?

(a)  $4x^2 - 3x + 2$

(b)  $x + \frac{2}{x}$  (c)  $\frac{1}{2}t + 3t$  (d)  $2y^3 + 3y$

(9) Home Work :-

In the following, identify the monomials, binomials and Trinomials -

(i) (a)  $y^2$

(b)  $m^2 + 2m$

(c)  $+6 - \frac{5}{3} + 3 + 6$  (d) 7 (e)  $7u^6$

(ii)  $3y^2 - 2y + 4$  is it a polynomial in y?

(iii) Explain ascending and descending.

- 55 -

## LESSON PLAN - 2

NAME : MR. J. H. KUREAH

TOPIC : SET THEORY : (Elementary Theory )  
For Class IX

One of the branches of Mathematics is SET. In general, a set is defined as a list, or a collection or a class of well-defined objects. The objects in Sets, can be anything : numbers, people, letters, rivers, towns, cities, materials, etc.

The followings are some of the examples of sets -

- i) The set of numbers 1, 3, 5, 7 to 10
- ii) The set of vowels in English alphabet: a, e, i, o, u
- iii) The students of Class IX: Tom, Dick, Paul, Rick
- iv) The set of countries in Europe, England, France, Spain, Germany
- v) The set of even numbers : 2, 4, 6, 8, ..... etc. etc.

We observe here that some of the sets of the above examples are listed by stating their properties, i.e. rules which decide whether or not a particular object is a member of set.

Notation: Sets are usually denoted by capital letters A, B, C, X, Y, Z, .....

The members in the sets are called the elements, which can be represented either by numerals, 1, 2, 3, 4, ..... or by small letters such as a, b, x, y, .....

As for example, we write  $A = \{1, 3, 7, 8\}$ . It means A is a set which consists 1, 3, 7, 8 as its elements. Here

- 1 is an element of set A,
- 3 is an element of set A,
- 7 is an element of set A, and
- 8 is an element of set A.

The above statements can be expressed in symbols also, as -

$1 \in A$ ,  $3 \in A$ ,  $7 \in A$  and  $8 \in A$ .

The symbol ' $\in$ ' means 'belongs to' or 'is an element of'.

We can also use different notations or forms in writing a set. Generally, there are two forms of writing

a set. The first form is called Roster form or Tabular form and the second form is called as Set Builder form.

In Roster form, we generally define a set by listing its members one by one, separated by commas and enclosing them in brackets or braces thus ' ' e.g.  $A = \{1, 2, 3, 4\}$ .

But if we define a particular set by stating its properties which its elements must satisfy; for example, let B be the set of all even numbers, then we use a letter, usually x, to represent an arbitrary element. We write as

$B = \{x : x \text{ is even}\}$  and we say that 'B is the set of numbers x such that x is even'. The notation ':' is read as 'such that'.

This second form of writing a set is called the Set Builder Form.

As indicated above the symbol 'E' means 'belongs to' and if the element does not belong to the set, we use ' $\notin$ ' which means 'does not belong to' e.g.  $M = \{1, 3, 4\}$ . Here  $2 \notin M$ .

More examples we can give

$$A = \{5, 10, 15, 20\}$$

$$B = \{a, e, i, o, u\}$$

$$C = \{-3, 3, 2, -2, 1, -1, 0\}$$

$$D = \{x : x^2 - 3x - 2 = 0\}$$

$$E = \{x : x \text{ is a capital and } x \text{ is a city in India}\}$$

$$F = \{x : x \text{ is name of a month in a year}\}$$

ect. etc.

### Finite and Infinite Sets

Sets can be finite or infinite. A set is finite if it consists of a specific number of elements i.e. in counting the different members of the set, the counting process can come to an end.

e.g. (i)  $M = \{x : x \text{ is the day of the week}\}$

This is a finite set

(ii)  $P = \{2, 3, 5, 7, \dots\}$

This is an infinite set

### Equality of Sets

Set 'A' is equal to the set 'B' if every element which belongs to A also belongs to B and if every

element which belong to B also belong to A. We denote this equality of sets A and B by writing  $A = B$ .

e.g.  $A = 1, 2, 3, 4$        $B = 4, 1, 3, 2$

Then  $A = B$ . Here we notice that a set does not change if its elements are re-arranged.

### NULL SET

We may sometimes come across the concept of an empty set, i.e. a set which contains no elements. This set is sometimes called as the Null Set, a void set or an empty set. Symbolically, we denote the empty set as

' $\{ \}$ ' or ' $\emptyset$ '

e.g.  $A = \{x : x \text{ is a letter before 'a' in the alphabet}\}$

$B = \{x : x \text{ is odd number and } x \leq 1\}$

## LESSON PLAN -3

### SET THEORY

NAME : MR. EVERARD D. NONGSIANG

CLASS VII

A set is defined as a collection of things or objects. For example - a dinner set, a set of benches, desks, etc.

For the set of objects, there is always the greatest set and it is called the universal set. The universal set is represented by a rectangle

u

The sets are generally denoted by the capital letters like A, B, C, .....

The set is represented by a circle within the rectangle

(A)

A set consists of objects which are called elements of the set. They are denoted by the letters like a, b, c, .... let A be any set and a is an element; we write as 'a belongs to A

is  $a \in A$

$\in$  means 'belong to'

$x \notin A$ , ' $\notin$ ' means 'does not belong to'

The sets of months in the year begins with J

$A = \{ \text{January, June, July} \}$ , we say

January  $\in A$ , June  $\in A$ , July  $\in A$ .

#### Null or Empty Set

A set having no element is called a Null or Empty Set and it is denoted by  $\emptyset$ .

A set having only one element is called a Singleton Set. If a is an element of a singleton set then it is written as  $A = \{ a \}$ .

#### Finite Set

A set having a finite number of elements is called a finite set.

$A = \{x, x_1, x_2, x_3, \dots, x_n\}$  is a finite set containing  $n$  elements.

### Equal Sets

Two sets are equal if they have the same elements

e.g.  $A = \{a, b, c, d\}$        $B = \{c, d, b, a\}$

(1) e.g.  $A = \{a, b, c\}$

$B = \{x, y, z\}$  In the example (i) for each element in a given set, there is a corresponding element in another set. So there exists a 1 to 1 correspondence between the elements of both sets. This is called an equivalent set.

Two equal sets are always equivalent.

But two equivalent sets are not always equal

e.g.  $A = \{a, b, c, d\}$        $B = \{1, 2, 3, 4\}$

$A = \{a, b, c, d\}$        $B = \{1, 2, 3, 4\}$

### Sub Sets

Let  $A$  and  $B$  any two sets. We say that  $A$  is a sub set of  $B$  if all the members of  $A$  also belong to  $B$ , is denoted as  $A \subset B$ .

e.g.  $A = \{a, b, c\}$

$B = \{a, b, c, 1, 2\}$

$A \subset B$



If  $A$  is a subset of  $B$ , but are not equal, we say that  $A$  is a proper subset of  $B$  and we write  $A \subset B$ .

## LESSON PLAN -4.

MRS. T. WINFAR LANONG LALONG

CLASS - VII

TRIGONOMETRY

1. GENERAL AIM : To introduce Trigonometry to the students which is totally a new branch of Mathematics.
2. SPECIFIC AIM : To inculcate the ideas of trigonometrical ratios and to familiarise them with its mode of calculation.
3. PREVIOUS KNOWLEDGE : The students have the knowledge of the ratios, sides of a triangle.
4. PRESENTATION : Trigonometry is an important branch of Mathematics. The word trigonometry is derived from three Greek roots: 'trio' meaning 'thrice', 'gonia' means an angle and 'metron' means measure. Thus trigonometry is the study of a three-sides figure i.e. a triangle.

The Study of Trigonometry is of great importance in Surveying, Astronomy, Navigation, Engineering etc.

Generally one of the following Greek letters or English alphabets is used as symbols to denote an angle.

$\alpha$  - alpha

$\beta$  - Beta

$\gamma$  - Gamma

$\theta$  - Theta

$\phi$  - Phi

$\psi$  - Psi

and A, B, C etc.

Let a line OP make an angle  $\angle XOP = \theta$  with OX. From any point A on OP, a perpendicular AB is dropped on OX. Then w.r.t. angle  $\theta$  AB is the perpendicular or opposite side (p), BC, the base or adjacent side (b), and AC, the hypotenuse (h)

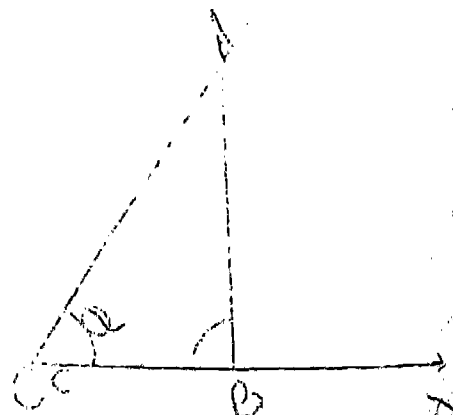


Fig.1

Out of these three sides, six ratios can be formed, namely,  $\frac{p}{h}$ ,  $\frac{b}{h}$ ,  $\frac{p}{b}$ ,  $\frac{h}{p}$ ,  $\frac{h}{b}$ , and  $\frac{b}{p}$ .

These ratios are called the Trigonometrical ratios.

The ratio	$\frac{p}{h}$	is called the sine of $\angle C$
" "	$\frac{b}{h}$	" " " cosine of $\angle C$
" "	$\frac{p}{b}$	" " " tangent of $\angle C$
" "	$\frac{h}{p}$	" " " Cosecant of $\angle C$
" "	$\frac{h}{b}$	" " " Secant of $\angle C$
" "	$\frac{b}{p}$	" " " Co-tangent of $\angle C$

In short, we write

$$\begin{aligned} \frac{p}{h} &= \sin \angle C, & \frac{h}{p} &= \operatorname{Cosec} \angle C \\ \frac{b}{h} &= \cos \angle C, & \frac{h}{b} &= \sec \angle C \\ \frac{p}{b} &= \tan \angle C, & \frac{b}{p} &= \cot \angle C \end{aligned}$$

## 5. RECAPITULATION :

The students are asked to define the trigonometrical ratios and to name the different sides of a triangle and to name or identify the different trigonometrical ratios.

Illustrated Examples :-

- (a) In the adjoining diagram  
 $AB = 3$  cm,  $BC = 4$  cm and  
 angle  $B = 90^\circ$ . Calculate  
 all trigonometrical ratios  
 of angle  $C$ .

By pythagoras theorem

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ &= 3^2 + 4^2 \\ &= 9 + 16 \\ &= 25 \end{aligned}$$

$$\therefore AC = \sqrt{25}$$

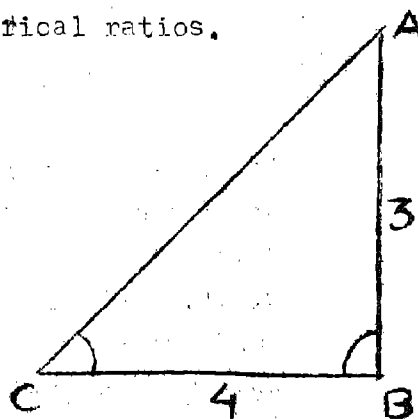
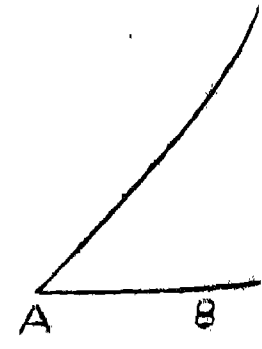
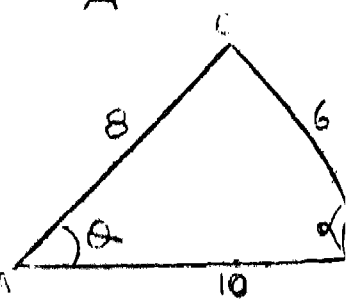
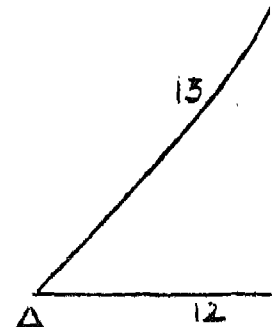


Fig.2

$$\begin{aligned}\therefore \sin C &= \frac{p}{h} = \frac{3}{5} \\ \cos C &= \frac{b}{h} = \frac{4}{5} \\ \tan C &= \frac{p}{b} = \frac{3}{4} \\ \operatorname{cosec} C &= \frac{h}{p} = \frac{5}{3} \\ \sec C &= \frac{h}{b} = \frac{5}{4} \\ \text{and } \cot C &= \frac{b}{p} = \frac{4}{3}\end{aligned}$$

#### 6. HOME TASKS

- Write down the values of (i)  $\sin A$  (ii)  $\cos A$ , (iii)  $\tan C$  (iv)  $\operatorname{cosec} C$  (v)  $\sec A$ , (vi)  $\cot A$ .
- From the adjoining figure, find,  $\sin Q$ ,  $\cos Q$ ,  $\tan Q$ ,  $\sin \alpha$ ,  $\cos \alpha$ ,  $\tan \alpha$ .
- Write down the values of :  
(i)  $\sin A$  (ii)  $\cos A$  (iii)  $\tan C$ ,  
(iv)  $\operatorname{cosec} C$ , (v)  $\sec A$ , (vi)  $\cot A$   
(Using the given diagram).
- Write down the value of  
(i)  $\sin A$  (ii)  $\cos A$  (iii)  $\tan$   
(iv)  $\operatorname{cosec} C$  (v)  $\sec A$  (vi)  $\cot A$   
(Using the given diagram).



LESSON PLAN-5

MRS. IARLIMON D.BLAH

DATE	: 16.3.93	CLASS	: VI
SUBJECT	: MATHEMATICS	AVERAGE AGE OF PUPILS	: 11 year
TOPIC	: SIMPLE INTEREST	DURATION	: 45minut

AID MATERIAL NEEDED

- (1) Blackboard, duster, chalk, painter etc.
- (2) A chart showing a person depositing some amount in a bank an withdrawing it after 3 years.
- (3) The Bank pass book of the person showing his account.
- (4) A chart showing interest rates of different banks, agencies and post offices of a town.

GENERAL AIM :

- (1) To help the pupils to acquire mathematics knowledge of terms, concepts, symbols, definitions and principles processes of mathematics.
- (2) To help the pupils to understand terms, concepts, symbols etc.
- (3) To help the pupils to acquire skills in computation, reading tables, charts etc.
- (4) To develop the ability to apply their knowledge and understanding of mathematics to unfamiliar situations.
- (5) To help the pupils to appreciate the role of mathematics in day to day life.

PRESENTATION

MAJOR IDEA	CONTENT OUTLINE	LEARNING SITUATION/ ACTIVITY	COMPETENCY
1. Concept of Interest.	(i) Supposing we deposit money in bank or post office in the Saving Bank account. Since the bank or Post Office may	Suppose Sita deposited Rs.10,000/- in the bank Account. The bank pays interest at 6% per annum. What interest should Sita get at the end of	1. The pupils will be competent to understand the concept of interest

keeps the money 2 years?  
it gives us in- (1) What amount  
terest. would she get  
in all after 2  
years?

and they  
will know  
the terms  
such as  
Principal,  
Amount, In-  
terest.

Interest-Is the additional money the pass book  
changed for its after 2 years  
use i.e. (Monthly will be seen.  
rent or borrow- (2) How much amount  
ed is called In- does it give in.  
terest. terest after 2  
years.

(ii) The amount It will show that  
lent on borrow- after 2 years the  
ed is called amount will be  
Principal. Rs. 11,200/-.

(iii) Amount - (3) What is this  
Amount due at additional money  
that time is the of Rs. 1200/-. This  
total money we additional money  
receive or pay, is the interest  
if the interest paid by the bank  
due is added to at the end of 3  
the principal. years.

Amount = Prin- (4) The total  
cipal + Inte- amount written in  
rest. the pass book af-  
ter 2 years  
= 11,200/-.

This Rs. 11,200/-  
is the Amount.  
Amount = Princi-  
pal + Interest  
Rs. 11,200/- = 10,000/-  
+ Rs. 1,200/-.

## 2. Concept of Simple Interest.

(i) There are two types of interest-simple interest and compound interest.

(ii) Simple Interest is calculated only on the principal initially invested for each year of its use.

This Rs. 1,200/- is the simple interest, because it is calculated only on the principal initially invested i.e. Rs. 10,000/- for 2 years of its use.

2. They will know and understand the meaning of simple interest.

## 3. Rate of Interest.

(i) The interest is paid according to an agreement which is in the form of a note (R) per unit

As the bank offers interest of 6% per annum on Saving Bank Account. So it means that the interest on Rs. 100/-

of the principal for each year's use lent or borrowed is Rs.6/-.

(ii) Rate of interest is given in the form of a percent of the principal per year per annum.

Suppose a bank offers interest of Rs.6% per annum.

(1) What does it mean?

It means that 6% per annum means that the interest on Rs.100 for each years use is Rs.6.

4. Factors determining interest.

(1) Interest depends on three factors namely-

1. Principal,
2. Rate of Interest,
3. Period of time.

4. To explain the factors determining interest we can show the prepared table of a bank as follows having the following in the black-board.

4. The pupil will be able to understand the three factors on that interest depends i.e.

1. Principal,
2. Rate of Interest,
3. Period of Time.

#### LEARNING SITUATION/ACTIVITY

#### COMPETENCY

Depositor's Name	Money deposited (Principal)	Rate of Interest	Time for which money deposited.	Interest
1. Henry	Rs.10,000	6%	2years	Rs.1,200/-
2. Lama	Rs.20,000	6%	2years	Rs.2,400/-
3. Henry	Rs.100/-	5%	4years	Rs.20/-
4. John	Rs.100/-	6%	4years	Rs.24/-
5. Linda	Rs.3,000	7%	2years	Rs.420/-
6. Sam	Rs.3,000	7%	3years	Rs.630/-

By asking the pupils to study the above chart, we can ask the students to compare No.1 & No.2

- (1) Who got more interest?
- (2) And Why?

Because Rama deposited more money, so interest depends on the money deposited i.e. on the Principal.

While comparing No.3 + No.4

Who get more interest? And why?

Because No.4 i.e. John invested money at a higher rate of interest.

Thus interest depends on the rate of interest.

Asking pupils to compare No.5 + No.6

We can ask questions like

- (1) Who got more interest? And why?

Because Sam deposited the money for a longer period. From this we can conclude that Interest depends on Principal, rate and time.

5. Calculating Interest.

1. In calculating Interest we are applying the idea of percent and unitary method.

2. Then we evolve a chart method of calculating interest i.e.

$$\text{Interest} = \text{Principal} \times \text{Rate} \times \text{Time}.$$

In order to find out the amount we must remember that:

$$\text{Amount} = \text{Principal} + \text{Interest}.$$

EXAMPLE-I - Let us do some calculation work on simple interest and see whether all the concepts related to simple interest are clear or not.

Q.1. Suppose Sita deposited Rs. 20,000 in a bank. If the bank pays interest at 14% per annum, determine the interest and also the amount Sita will get after 5 years.

Hence Principal = Rs. 20,000/-  
Rate = 14% per annum  
Time = 5 years

Q.1. What will you do to know the interest the bank will pay at the end of 5 years?

So using formula i.e. Interest = Principal x Rate x Time.  
The simple Interest = Rs. (20,000/- x 14 x  $\frac{5}{100}$ ) = Rs. 15,400/-

∴ Interest Sita will get Rs. 15,400/-. As they have already learn to that:

$$\text{Amount} = \text{Principal} + \text{Interest}$$

$$\therefore \text{Amount} = \text{Rs. } 20,000/- + \text{Rs. } 15,400/- = \text{Rs. } 35,400/-$$

Thus, the bank paid to Sita at end of a stipulated period Rs. 15,400/- as interest. The total amount paid = Rs. 35,400/- by bank to Sita.

5. The pupils will be competent to discover formula and method used for the calculation of simple interest.

**RECAPITULATION:** The teacher briefly recapitulates i.e. going over the whole lesson after it has been taught, to enable the pupils to methodically review, what they have just learned and give an opportunity to pupils to ask questions about things

they have not understood.

HOME ASSIGNMENT:- The pupils will be asked to do some home works and classworks to calculate simple interest by giving some exercises related to the topics.

1. Find the Interest:-

- (i) On Rs.100/- for  $2\frac{1}{2}$  years at 7% per annum
- (ii) On Rs.2,200/- for 6 months at 3% per annum.

2. Sita borrows Rs.400/- from her uncle. She agrees to pay it back after one year together with the interest of 4% per annum? What amount will she pay back?

APPLICATION:- (1) By knowing the concept of Simple Interest, the pupils will know and understand how to read the chart showing the rates of interest offered by different banks, agencies etc.

(2) They will know also how to calculate the interest on the amount they invested or deposited on banks at different rates of interest and time. They will also know how to calculate the interest they have to pay to the money lenders on banks, in case they are taking some money as loans from money lenders or banks.

(3) They will develop interest in mathematics, as they can use of their knowledge in practical.

LESSON PLAN - 6

NAME : SHRI.K.W.WANKHAR

CLASS : VII

TOPIC : RATIONAL NUMBERS : (Fractions & Integers as Rational Numbers)

In previous classes we have learnt about natural numbers and whole numbers and also about integers and fractions. We have already studied that the numbers 1,2,3,4.... so on in our earlier classes are natural numbers. Similarly, 0,1,2,3.....so on are whole numbers. We also can recall that the sum of two fractions is always a fraction but it is not always possible to subtract a given fraction from another fraction. (e.g. - can you subtract  $2\frac{2}{5}$  from  $1\frac{3}{5}$ ?). Similarly, the product of two integers is always an integer, but it may always be possible for a given integer to exactly divide another given integer (e.g. Does -2 divide -5?).

Thus, we therefore need to extend our number system so that it may be possible to subtract a given number by another given number different from Zero. (Noted that division by Zero is not possible).

We can therefore in the language of equations, extend the number system in such a way that equations such as  $2x + 5 = 0$ ,  $3x + 7 = 0$  which do not have any solution in the system of integers or in the system of fractions but will have solution in this new rational number system.

Consider the fractions, i.e. the numbers :-

$\frac{0}{1}, \frac{1}{1}, \frac{1}{2}, \frac{2}{2}, \frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \dots\dots\dots$

Corresponding to each fraction we can form new numbers by replacing its numerator or denominator of both by its negative sign.

e.g. Suppose -

(i) Corresponding to the fraction  $\frac{2}{3}$  we can form new number as  $\frac{-2}{3}, \frac{2}{-3}, \frac{-2}{-3}$

(ii) Corresponding to the fraction  $\frac{3}{3}$ , we can form new numbers as  $\frac{-3}{3}, \frac{3}{-3}, \frac{-3}{-3}$

(iii) Also, corresponding to the fraction  $\frac{0}{2}$ , we can form new number  $\frac{0}{-2}$  (There is no negative 0).

So, all numbers formed in this manner together with all the fractions are called RATIONAL NUMBERS.

Generalising the result, we have by saying that "A number which can be expressed in the form  $\frac{p}{q}$ , where 'p' and q are integers and  $q \neq 0$  (q is different from zero) is called a Rational Number".

So we concluded that;

- (1) All fractions are rational numbers
- (2) All integers are also rational numbers

Rational numbers are of great importance in our daily life situations. We use these numbers to represent profits and losses in business, altitudes of places with reference to sea level, temperatures above and below the standard temperature and so on and so forth.

Let us consider the following statements or understanding of Rational Numbers :-

(1) Fractions as Rational Numbers:-

The fraction  $\frac{3}{4}$  is expressed in the form  $\frac{p}{q}$  where  $p = 3$  and  $q = 4$ . Since every number of the form  $\frac{p}{q}$  where p, q are integers and  $q \neq 0$  is a rational number, therefore,  $\frac{3}{4}$  is a rational number. Similarly, the fraction  $\frac{8}{3}, \frac{2}{11}, \frac{32}{12}$  are all rational numbers.

Thus if 'x' and 'y' are positive integers, the fraction  $\frac{x}{y}$  is a rational number.

(2) Integers as Rational Numbers:-

Let us consider the fraction  $\frac{5}{1}$ . This fraction is the same as the integer 5. Similarly, the fraction  $\frac{-9}{1}$  is the same as the integer -9; Also  $\frac{-4}{1}$  is the same as -4. Thus for fractions  $\frac{5}{1}, \frac{-9}{1}, \frac{-4}{1}$  we can write the rational number  $\frac{5}{1}, \frac{-9}{1}, \frac{-4}{1}$ , which are respectively equal to 5, -9 and -4 and so on. Thus we say that "If x be any integer, the rational number  $\frac{x}{1}$  is the same as the integer 'x' which usually satisfies the rational number in the form  $\frac{p}{q}$  where p, q are integers and  $q \neq 0$ ".

(3) Positive & Negative Rational Numbers:-

We have learnt in earlier classes that if we multiply the numerator and denominator of a fraction by the same positive integer, the value of the fraction does not

change. The fraction  $\frac{1}{2}$  and  $\frac{2}{4}$  are equal because the numerator and denominator of  $\frac{2}{4}$  can be obtained by multiplying the numerator and denominator of  $\frac{1}{2}$  by 2.

Similarly,

$$\frac{-2}{3} = \frac{(-2) \times (-1)}{3 \times (-1)} = \frac{-2}{-3} = \frac{2 \times 2}{(-3) \times 2} = \frac{4}{-6} = \frac{4 \times (-2)}{(-6) \times (-2)} = \frac{-8}{12}$$

In the above examples of rational numbers

$$\frac{-2}{3}, \frac{2}{-3}, \frac{4}{-6}, \frac{-8}{12}$$

Consider, the rational number  $\frac{-3}{4}$

$$\frac{-3}{4} = \frac{(-3) \times (-1)}{4 \times (-1)} = \frac{3}{-4} = \frac{3 \times 2}{-4 \times 2} = \frac{6}{-8} = \frac{6 \times (-3)}{-8 \times (-3)} = \frac{-18}{24}$$

$$\therefore \frac{-3}{4} = \frac{3}{-4} = \frac{6}{-8} = \frac{-18}{24}$$

#### (4) Zero Rational Number

Every integer is a rational number, therefore the integer 0 is also a rational number. Thus,  $\frac{0}{1}, \frac{0}{-1}, \frac{0}{2}, \frac{0}{-2}, \frac{0}{3}, \frac{0}{-3}$  and so represents the rational number of Zero value which are all equal. That is they represent the same rational number of Zero value.

(Note :- to identify student -0 never exist).



LESSON PLAN - 8

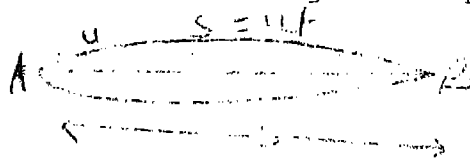
NAME - SHRI. WILLINGSON SUKHLAIN

ON EQUATION OF MOTION

For teaching the 'Equation of Motion', the teacher will mention the following four equations :

- (1)  $s = ut$  where  $s$  is the distance covered in  $t$  sec,
- (2)  $v = u + at$   $u$  - average velocity / uniform velocity
- (3)  $v^2 = u^2 + 2as$  final velocity
- (4)  $s = ut + \frac{1}{2}at^2$   $a$  = acceleration.

Then, each equation is to be derived and explained with their definitions. Starting from equation (1).



The particle/body travels with a uniform velocity ' $u$ ' and covers a distance in time ' $t$ ' i.e., takes  $t$  secs to travel from A to B. The distance between A and B is  $S$  (suppose). As we know (explain to them).

The distance travelled in time  $t$  = uniform velocity  $\times$  time taken

$$\text{i.e. } S = ut$$

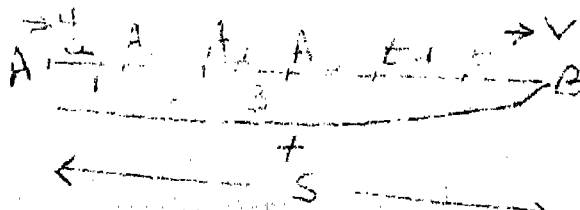
Uniform velocity is that velocity with which the body travels over equal distance in equal interval of time.

Each interval of time is 2 sec and the body travels is 4 m. (Shown in the figure)

If the body does not travel with uniform velocity, then this equation is not applicable or it is of the other form.

Few exercises are to be given in order that the student can understand and apply the above equation.

$$(2) \quad v = u + at$$



Definitions of initial, final velocity, acceleration, are to be given and conceptions of uniform accel<sup>n</sup>, uniform retardation are to be explained.

Suppose a particle at A starts its motion with initial velocity and is moving with a uniform accel<sup>n</sup> 'a' and reaches the point B with a velocity V, covers a distance S after a time interval t sec.

By equation (1)

Final velocity V = Initial velocity 'u' + ave in 'a' (at a<sub>1</sub>)

i.e.  $V = ut + a$  (after 1 sec at A<sub>1</sub>)

i.e.  $V = ut + 2a$  (after 2 secs at A<sub>2</sub>) so on

$V = ut + at$  (after 't' sec at B)

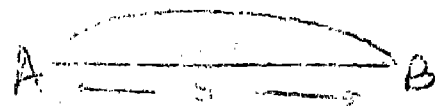
This equation is applicable only when acceleration is uniform rate of change if velocity is equal in equal interval of time.

Let a particle describe a distance in time 't' and let U and V be the initial and final velocities at the beginning and at the end of time 't'.

∴ The average velocity is  $U = \frac{u + v}{2}$  (A)

From equation (2)  $V = u + at$

or  $t = \frac{v - u}{a}$



Distance = Speed x time

$$\therefore S = \frac{(v + u)}{2} \times \frac{v - u}{a} \text{ (Substituting the value of t)}$$

$$= \frac{v^2 - u^2}{2a}$$

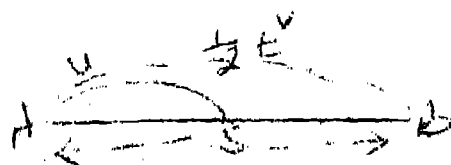
$$\text{or } 2as = v^2 - u^2$$

$$\text{or } v^2 = u^2 + 2as$$

$$\text{or } v^2 = u^2 - 2as \text{ when there is uniform retardation.}$$

Def<sup>n</sup> of average velo is to be given and explained to the students.

$$(4) S = ut + \frac{1}{2} at^2$$



Let  $V$  be the velo. at time  $\frac{t}{2}$ . The velocity  
 $V = u + at/2$  (  $u$  - initial velo.

$v$  - velo. at the end of  $t/2$  sec)

$a$  - acceleration

So,  $V$  can be considered as average velocity because it decreases and increases to the left and right at a uniform rate. So the distance ' $s$ ' covered in the whole time  $t$  is

$$s = vt$$

$$= (u + at/2) t$$

$$S = ut + \frac{1}{2}at^2 \quad \dots\dots\dots (B)$$

Some exercises involving this equation should be given to the students. Some basic questions should also be asked to know the effectiveness of teaching learning process.

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# LESSON PLAN - 102

NAME : MRS. B. LONGSHIANG  
CLASS : VI  
SUBJECT : ARITHMETIC

## TOPIC - SIMPLE INTEREST BY THE UNITARY METHOD

General Aim - To find the simple interest when rate percent per annum is given.

Specific Aim - To enable the pupils to discover the formula for finding the Simple Interest on Principal P, and T years at R %.

$$\text{viz } SI = \frac{\text{Principal} \times \text{Time} \times \text{Rate}}{100}$$

and to teach them to solve problems which may occur in their daily life, speedily and accurately.

### Previous Knowledge:-

The pupils can solve problems by Unitary method and have the understanding of the terms and processes involved. They are already familiar with Multiplication and Division.

Introduction:- A discussion with the pupils on saving bank accounts and how interest is reckoned. The following questions will be asked. (S.I.).

- (1) If Simple Interest (S.I) on Rs.100/- for 1 year is Rs.5/-, what will be the S.I. for Rs.1/- for 1 year?
- (2) In this case how will you calculate the S.I. for Rs.200 for 3 years? (Ans.  $\frac{200 \times 5 \times 3}{100}$  or  $\frac{P \times R \times T}{100}$ )

### Presentation :-

Matter	Method
<p>We know that Interest or S.I. = <math>\frac{\text{Principal} \times \text{Time} \times \text{Rate}}{100}</math></p> <p>we will solve the above problem with the help of the above formula. The formula may be represented by</p> <p><math>\frac{P \times T \times R}{100}</math> 4 represents the rate %, 4.p.c. per annum means Rs.4 is the interest on every Rs.100 for 1 year. Thus '4.p.c.' means 4.p.c per annum.</p> <p>Hence the Principal is Rs.400 Time = 2 years Rate = 4 p.c.</p>	<p>The solution of the following example will be elicited from the pupils, and written on the Blackboard by the teacher. Find the simple interest on Rs.400/- at 4.p.c. per annum for 2 years.</p> <p>The teacher while teaching will ask many questions to the students step by step and write the answers of the students on the B.B. The following question will be asked:-</p> <p>What is 4 here?</p> <p>Hence what is the interest for 1 year for Rs.100?</p>

What is the Principal according to the question?

What is the length of time? How much Interest you will get at the end of the two years?

- (a) Multiply the numerator.
- (b) Divide the numerator with the denominator.

So substituting the formula with the figures we have in the question -S.I.

$$= \frac{\text{Principal} \times \text{Time} \times \text{Rate}}{100}$$

$$= \text{Rs.} \frac{400 \times 2 \times 4}{100} = \text{Rs.} 32/-$$

Simple Interest = Rs. 32/-

From the above example, we have the following Rule

Rule- To find the Simple Interest when the rate percent is given. Multiply the Principal by the number of years and the rate percent and divide the result by 100.

Recapitulation and Application

Example No. 3, exercise 68.  
A new method arithmetic.

The pupils will be shown how to solve an example with the aid of this formula; the solution will be elicited from the pupils and written on the B.B. by the teacher.

For example, a sum will be solved.

What is S.I. of Rs. 550/- at the rate of 4.P.c. for 4 years?

$$\text{S.I.} = \frac{P \times T \times R}{100} = \frac{550 \times 4 \times 4}{100}$$

$$= \text{Rs.} 88/-$$

# LESSON PLAN 10

NAME : SHRI VICTOR NONGSIANG  
 SUBJECT : MATHEMATICS  
 TOPIC : 'PROFIT AND LOSS'  
 SUB TOPIC: 'GAIN' PERCENT'

## Meaning of the Terms:-

Cost Price, Selling Price, Actual Gain, Actual Loss and Gain/Loss Percent.

Before giving the meaning of the above terms, the teacher gives the following examples:-

1. Examples :- I bought an article for Rs.50/- and sold the same for Rs.60/-. What extra money I get?

From This example,

The money I have to pay =Rs.50/-

The money the customer will pay =Rs.60/-

Extra money I have to get =Rs.60- 50

=Rs.10

Here, Rs.50 is the Cost Price, Rs.60 is the Selling Price and Rs.10 is the Actual Gain.

2. Example :- Hari purchased an article for Rs.200 but due to some defects, he had to sell it for Rs.180. How much money he had to bear?

Here Rs.200/- = C.P.

Rs.180/- = S.P.

Extra money to be born = Rs.(200-180)

= Rs.20/-

∴ The extra money that Hari or the dealer had to bear is the actual loss.

From example -1, we find that

Out of Rs.60, actual gain = Rs.10

$$\therefore \frac{10}{60} = \text{Rs.} \frac{10}{60}$$

$$\therefore \frac{10}{60} \times 100 = \text{Rs.} \frac{10 \times 100}{60}$$

$$= \text{Rs.} \frac{50}{3}$$

$$= \text{Rs.} 16\frac{2}{3}$$

This Rs.16 $\frac{2}{3}$  which is gained out of Rs.100 is called the gain percent.

OR

Gain % = Gain out of 100

Similarly from example 2:

$$\begin{aligned} \text{Out of Rs. } 200, \text{ actual loss} &= \text{Rs. } 20/- \\ \therefore \frac{20}{200} \times 100 &= \text{Rs. } 10\% \\ \therefore \frac{20}{200} \times 100 &= \text{Rs. } 10\% \end{aligned}$$

This Rs. 10/- which is lost out of Rs. 100 is called the loss percent.

OR

Loss % = Loss out of 100

From (i) and (ii), We find that

$$\text{Gain \%} = \frac{\text{Actual Gain} \times 100}{\text{Cost Price}}$$

$$\text{Loss \%} = \frac{\text{Actual Loss} \times 100}{\text{Cost Price}}$$

Application:-

1. The cost price of an article is Rs. 200. It is sold for Rs. 250, find the gain percent.

C.P. = Rs. 200, where c.p.  $\Rightarrow$  cost price.

S.P. = Rs. 250, where s.p.  $\Rightarrow$  selling price

$$\therefore \text{Actual gain} = \text{Rs. } (250 - 200)$$

$$= \text{Rs. } 50$$

$$\therefore \text{Gain P.C.} = \frac{50}{200} \times 100$$

$$= \frac{50}{200} \times 100$$

$$= \text{Rs. } 25\%$$

$$= \text{Rs. } 25\%$$

2. The article which cost Rs. 50 had to be disposed off at Rs. 40, find the loss percent?

C.P. = Rs. 50, where C.P.  $\Rightarrow$  Cost Price

S.P. = Rs. 40, where S.P.  $\Rightarrow$  Selling Price

$$\therefore \text{Actual loss} = \text{Rs. } (50 - 40)$$

$$= \text{Rs. } 10$$

$$\therefore \text{Loss P.C.} = \frac{10}{50} \times 100$$

$$= \frac{10}{50} \times 100$$

$$= \text{Rs. } 20\%$$

LESSON PLAN -11

NAME : MRS. M. HOOROO

CLASS - VII, SECTION - A

No. of Pupils - 52.

TOPIC : 'MULTIPLICATION OF BINOMIAL SPECIAL PRODUCTS'

Teacher : Good morning everybody

Pupils : Good morning teacher

Teacher : Today we are going to study some special products, when we multiply binomials. In the last lesson, we learnt the multiplication of two binomials, say  $(a+b)$  and  $(c+d)$ , where the distributive property of multiplication over addition was used twice as follows:-

$$\begin{aligned}(a+b)(c+d) &= ax(c+d) + bx(c+d) \\ &= (ac+ad) + (bc+bd) \\ &= ac+ad+bc+bd\end{aligned}$$

Do you recall what this type of procedure is known as?

Expected Response) It is known as the horizontal method

Teacher : There is another method too, what do you call it?

Ex. Response: The second method is called the column method

Teacher : There are some special products where we will use the multiplication of binomials. They are:-

I.  $(a+b)(a+b)$  or,  $(a+b)^2$

II.  $(a-b)(a-b)$  or,  $(a-b)^2$

III.  $(a+b)(a-b)$

Let us consider the first expression:-

$$\begin{aligned}(a+b)^2 &= (a+b)(a+b) \\ &= a(a+b) + b(a+b)\end{aligned}$$

What is the property used here?

Expected Response : The distributive property

Teacher : Good. So we have :-

$$\begin{aligned}&= (a^2+ab) + (ba+b^2) \\ &= a^2+ab+ab+b^2 \\ &= a^2+2ab+b^2 \quad (\text{by using the commutative property and adding the like terms}).\end{aligned}$$

Thus we have :-

$$(a+b)^2 = a^2 + 2ab + b^2$$

Suppose we have the following expression:-

$$(x+y)^2$$

What will be the product?

Copy it down and try in your exercise books. You can do any one of the two methods, either by the horizontal or the column method.

Expected : We have  $(x+y)^2 = x^2 + 2xy + y^2$   
Response

Teacher : Again, try this other expression

$$(m+n)^2$$

Ex. Response:  $(m+n)^2 = m^2 + 2mn + n^2$

Teacher : So, we find that whatever be the terms of the binomial, the square of the binomial is equal to the sum of the squares of the first and the second terms together with twice the product of the first and second terms. In other words, the square of the binomial is equal to the square of the first term plus the square of the second term plus twice the product of the first term and second term.

Pupil : Is this expression  $(a+b)^2 = a^2 + 2ab + b^2$  an equation?

Teacher : Yes, it is an equation which has a special feature and, it is called an identity, because it is valid for all values of  $a$  and  $b$ . Now let us see the second expression,  $(a-b)^2$ .

We will have:-

$$\begin{aligned}(a-b)^2 &= (a-b)(a-b) \\ &= a(a-b) - b(a-b) \\ &= a^2 - ab - ba + b^2 \\ &= a^2 - 2ab + b^2\end{aligned}$$

Now you can try to find out the product of these expressions in your exercise books:

$$(x-y)^2;$$

$$(m-n)^2.$$

Ex. Res. :  $(x-y)^2 = x^2 - 2xy + y^2$

$$(m-n)^2 = m^2 - 2mn + n^2$$

Teacher : We can thus say that the square of a binomial of the form  $(a-b)^2$  is equal to the

square of the first term plus the square of the second term minus twice the product of the first term and the second term. Let us examine the third expression.

$$\begin{aligned}\text{We have: } (a+b)(a-b) &= a(a-b) + b(a-b) \\ &= a^2 - ab + ab - b^2 \\ &= a^2 - b^2\end{aligned}$$

Similarly, the product of

$$\begin{aligned}(x+y)(x-y) &= x(x-y) + y(x-y) \\ &= x^2 - xy + xy - y^2 \\ &= x^2 - y^2\end{aligned}$$

Hence the product of the sum and difference of any two same quantities (binomial) is equal to the difference of their squares. Can you find out what is the product of  $(m+n)(m-n)$ .

Ex. Response:  $(m+n)(m-n) = m^2 - n^2$

Teacher : Good

So you now have these relations

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2, \text{ and}$$

$$(a+b)(a-b) = a^2 - b^2$$

Such relations are called identities, because they are valid for all values of  $a$  and  $b$ .

$$\text{e.g. } (7+3)^3 = 7^2 + 2 \times 7 \times 3 + 3^2$$

$$= 49 + 42 + 9$$

$$= 100$$

$$(7-3)^2 = 7^2 - 2 \times 7 \times 3 + 3^2$$

$$= 49 - 42 + 9$$

$$= 16$$

$$(7+3)(7-3) = 7^2 - 3^2$$

$$= 49 - 9$$

$$= 40$$

$$\text{OR } (7+3)(7-3) = 7(7-3) + 3(7-3)$$

$$= (7 \times 4) + (3 \times 4)$$

$$= 28 + 12$$

$$= 40$$

We can use the identities directly in solving the following problems:-

(i)  $(2m+4n)^2$  (ii)  $(\frac{3}{4}p - 5q)^2$

(iii)  $(2u+3v)(2u-3v)$  (iv)  $(102)^2$

(v)  $99^2$  (vi)  $102 \times 98$

(All the above problems will be solved on the black-board by the teacher and the pupils will be asked to copy them in their exercise books).

Teacher : Will you be able to do the exercise following

Ex. Res. : Yes teacher

Teacher : Thank you everybody,

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LESSON PLAN -12

NAME: T. WINFAR LANONG LALOO

SUBJECT : ALGEBRA

LESSON : POLYNOMIALS

LESSON UNIT: FIRST LESSON ON POLYNOMIALS.

GENERAL AIM : To guide the pupils to understand clearly the new approach to Modern Mathematics and its utility in all aspects of life.

SPECIFIC AIM : To make the students have a clear concept on polynomials.

APPLIANCES : (1) Black Board (2) Chalks (3) Duster

PREPARATION : Previous knowledge : The students have the idea of the signs of operation, terms, variables, constants and co-efficients.

STEPS	Subject matter	Teacher's Work	Pupil's Work
P	To explain the meaning of 'Polynomial'	Firstly, before we come to the proper subject matter, let me explain what is meant by 'Algebraic expression'.	
R		An algebraic expression is a combination of terms which are connected by the signs of operation, namely, addition(+), subtraction(-), multiplication(x) and division(+).	
E		The following are a few examples of algebraic expressions.	
S		(i) $2a+3b$ (ii) $3m + \frac{4}{n^2}$	
N		(iii) $\frac{5x}{2} - \frac{6x^2}{2} + 3x$	
T		The word Polynomial means algebraic expression consisting of many terms involving powers of the variable, under the operation of addition and subtraction only.	
A		'Poly' means many.	
I		The examples of polynomials are:	
O		(i) $3x+4y+3$	
N			

- (ii)  $\frac{2}{3}x^2 + 4x + 3y + 8$   
 (iii)  $x+y+z+9$   
 (iv)  $\sqrt{2x} + \sqrt{3y} + 5$   
 (v)  $5m^2n - \frac{1}{3}mn + \sqrt{2}$

But it should be noted that

$\frac{1}{x}, x, \frac{1}{x+z}, x+4\sqrt{x} + \sqrt{y}$  are not polynomials as they involve operations of division and extraction of roots of the variables.

P General form  
of a polyno-  
R mial.

The general form of a polynomial is written as  $P(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$  where  $a_0, a_1, a_2, \dots, a_n$  are constants,  $n \in \mathbb{N}$ .

E  
S  
E Types of poly-  
nomials

Polynomials which contain only one term are called MONOMIALS. For example,  $2x, 2x^2, 3$  etc. are monomials. The pupils will try to give the examples of their own of the monomials.

N  
T  
A

Polynomials which contain only two terms are called BINOMIALS. BI means two.  $x+3, 2x^2+3y^3$ , etc. are examples of binomials. binomials, trinomials.

T  
I  
O

Polynomials having three terms are called TRINOMIALS. TRI means three. For example,  $x^2-5x+6, a+b+c$  are trinomials.

N

Degree of a  
polynomial

The highest exponent which appears in a polynomial is called the degree of the polynomial.

A polynomial of degree  $n$  is written as  $P(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$ ,  $n \in \mathbb{N}$

For examples,

i)  $x^2+x+1$  is a polynomial in  $x$  of degree 2.

ii)  $x^2+xy+y^2+x^2y$  is a polynomial in  $x$  and  $y$  of degree 3.

iii)  $x^5+x^3+2$  is a polynomial in  $x$  of degree 5.

The pupils try to give example of polynomials of different degree.

Polynomials of different degrees.

A polynomial of same degree is called a Homogeneous Polynomial.

For example,  $x^2y + x^3 + y^3 + x^3y$  is a polynomial of same degree (of degree 3).

A polynomial of degree two is called a Linear Polynomial and is denoted by  $P(x) = ax + b, a \neq 0$

A polynomial of degree two is called a Quadratic polynomial and is denoted by  $P(x) = ax^2 + bx + c, a \neq 0$

A polynomial of degree three is called a Cubic polynomial and a polynomial of degree four is called a Biquadratic polynomial.

P A few questions  
R on polynomials  
E and their different types.

(A) Which of the following are polynomials? (i)  $x^2 + x + 1$   
(ii)  $\sqrt{x^3} + 8$  (iii)  $y^3 - 5y + \sqrt{y}$

The pupils will answer these questions.

(B) Name the degree of the following polynomials

i)  $3x^4 - 4x^3 + 2x - 7$

ii)  $1 - y + y^2 - y^3 + y^4$

iii)  $5x^3 + 8x^2$

(C) Which of the following are monomials, binomials and trinomials?

i)  $3x + 5y$  (ii) 8 (iii)  $x^2$

(iv)  $x^3 + x - 7$

The students are asked to do the following questions for Home Works.

(1) Which of the following are Polynomials?

i)  $t^2 + \frac{1}{t^2}$  (ii)  $\frac{2}{3}x^2 + \sqrt{3}x + 1$

iii)  $3x^3 + 4x^2 - 5x + 10$  (iv)  $2\sqrt{x} + 3$

(v)  $\frac{1}{\sqrt{x}} + \sqrt{x}$  (vi)  $\frac{2}{x^2} + 5$

(2) Write down the degree of the following polynomials:

(i)  $3x^4 - 4x^2 + 2x - 7$

(ii)  $7x^2 + 2x$  (iii)  $xy + y^2 + x^2 + x^2y$

(iv)  $3x + 5$

(3) Which of the following are monomials, binomials or trinomials?

(i)  $z^2$  (ii)  $5x^2 + 8x + 1$

(iii)  $6x^3 + 5x^2$

(iv) 7 (v)  $5x + 3x$

(vi)  $x^2 + 4x + 2z + 9$

## PROBLEMS OF TEACHING MATHEMATICS

The teacher participants were taken into confidence for discussing the problems being faced in the teaching of Mathematics in classroom situation. The following problems were noted:-

1. Lack of interest and motivation towards learning mathematics.
2. Lack of teaching aids, charts, models, geometrical instrument mathematics kits etc.
3. Lack of proper environment of mathematics teaching.
4. Lack of qualified mathematics teachers in the schools. The old teachers fail to understand and grasp even some of the newly incorporated topics which cause their detachment from teaching.
5. The classes are heterogeneous. The students from various walks of life, having different aptitude, attitude and level of attainment come to constitute the very basis of classes. So the achievement differs and thus their performances are not appreciative and upto the mark.
6. The students lack study and learning habits. At the same time, many teachers do not involve themselves in proper study. Thus they fail to create a proper teaching learning situation.
7. The time schedule is so tight and compact that the teachers find unable to cope up. Even they are not in a position to finish the topic with full justification. Most of the students do not understand what is taught. Sometimes they claim it was beyond their understanding. The process leads to drop-out and wastage and stagnation at this level.
8. Many students do not get required amount of guidance at home which lead them not to complete their home task. Even they fail to understand the theme of the topics taught in the classes due to lack of guidance.
9. No remedial teaching is organised for the weaker students either through conducting territorial or extra classes.
10. Tution has come as a bolt between academic relationship of students and teachers. The teachers are so

occupied otherwise and thus have less time to share with their students. It has led to degeneration of teaching-learning atmosphere.

11. Less number of periods allotted for teaching of mathematics subject.

These are the aspects of problems which loom large on the teaching-learning scenario and thus the desired goals are not fulfilled.

#### SUGGESTIONS TO IMPROVE TEACHING AND LEARNING

During the discussion the following measures were suggested for improving the teaching of mathematics courses effectively.

1. The teachers should devote the whole heartedly towards fulfilling their obligations and get mastery over the subject. They should keep them in constant touch with the new developments in the area of school mathematics.
2. Teaching aids in the subject should be organised to create motivation, interest and clarify certain points. The teaching aids give thrust on learning front.
3. Mathematics teaching should be done in mathematics laboratory. Cassettes on mathematics teaching should be shown to acquaint the teachers with new techniques and the students to judge their weaknesses to short it out.
4. Refresher courses or inservice training should be conducted atleast once in a year or two, to enable the teachers to update their knowledge.
5. The guardians/parents should be requested to make their ward seated both times at home and try to do their home work. The educated guardians can help in doing their task but the uneducated only can seat with their sons and daughters to regulate their habit to seat and read.
6. Self teaching learning materials should be developed and distributed to the weak students so that they can read and understand themselves.
7. Maximum sums should be done in classroom. Classroom tasks should be checked regularly and correction should be suggested.
8. Remedial teaching for the weak students should be organised. Mathematics club should be formed in the schools. Mathematics quiz should be organised by the teacher to enable the students to participate and judge their knowledge and understanding. Book fair and mathematics fair should be organised to seek maximum involvement of the students.
9. Some more periods should be allotted for mathematics teaching in the classes so that maximum work can be done.

If these suggestions are given practical tone, certainly some changes in teaching can be brought.

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